CONTENTS

PROLOGUE: Principles of Problem Solving

1. Let r be the rate of the descent. We use the formula time $= \dfrac{\text{distance}}{\text{rate}}$; the ascent takes $\dfrac{1}{15}$ h, the descent takes $\dfrac{1}{r}$ h, and the total trip should take $\dfrac{2}{30} = \dfrac{1}{15}$ h. Thus we have $\dfrac{1}{15} + \dfrac{1}{r} = \dfrac{1}{15} \quad \Leftrightarrow \quad \dfrac{1}{r} = 0$, which is impossible. So the car cannot go fast enough to average 30 mi/h for the 2-mile trip.

2. Let us start with a given price P. After a discount of 40%, the price decreases to $0.6P$. After a discount of 20%, the price decreases to $0.8P$, and after another 20% discount, it becomes $0.8\,(0.8P) = 0.64P$. Since $0.6P < 0.64P$, a 40% discount is better.

3. We continue the pattern. Three parallel cuts produce 10 pieces. Thus, each new cut produces an additional 3 pieces. Since the first cut produces 4 pieces, we get the formula $f(n) = 4 + 3(n-1)$, $n \geq 1$. Since $f(142) = 4 + 3(141) = 427$, we see that 142 parallel cuts produce 427 pieces.

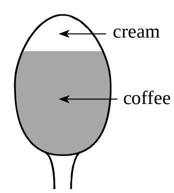
4. By placing two amoebas into the vessel, we skip the first simple division which took 3 minutes. Thus when we place two amoebas into the vessel, it will take $60 - 3 = 57$ minutes for the vessel to be full of amoebas.

5. The statement is false. Here is one particular counterexample:

	Player A	Player B
First half	1 hit in 99 at-bats: average $= \frac{1}{99}$	0 hit in 1 at-bat: average $= \frac{0}{1}$
Second half	1 hit in 1 at-bat: average $= \frac{1}{1}$	98 hits in 99 at-bats: average $= \frac{98}{99}$
Entire season	2 hits in 100 at-bats: average $= \frac{2}{100}$	99 hits in 100 at-bats: average $= \frac{99}{100}$

6. *Method 1:* After the exchanges, the volume of liquid in the pitcher and in the cup is the same as it was to begin with. Thus, any coffee in the pitcher of cream must be replacing an equal amount of cream that has ended up in the coffee cup.

Method 2: Alternatively, look at the drawing of the spoonful of coffee and cream mixture being returned to the pitcher of cream. Suppose it is possible to separate the cream and the coffee, as shown. Then you can see that the coffee going into the cream occupies the same volume as the cream that was left in the coffee.



Method 3 (an algebraic approach): Suppose the cup of coffee has y spoonfuls of coffee. When one spoonful of cream is added to the coffee cup, the resulting mixture has the following ratios: $\dfrac{\text{cream}}{\text{mixture}} = \dfrac{1}{y+1}$ and $\dfrac{\text{coffee}}{\text{mixture}} = \dfrac{y}{y+1}$.

So, when we remove a spoonful of the mixture and put it into the pitcher of cream, we are really removing $\dfrac{1}{y+1}$ of a spoonful of cream and $\dfrac{y}{y+1}$ spoonful of coffee. Thus the amount of cream left in the mixture (cream in the coffee) is $1 - \dfrac{1}{y+1} = \dfrac{y}{y+1}$ of a spoonful. This is the same as the amount of coffee we added to the cream.

7. Let r be the radius of the earth in feet. Then the circumference (length of the ribbon) is $2\pi r$. When we increase the radius by 1 foot, the new radius is $r + 1$, so the new circumference is $2\pi\,(r+1)$. Thus you need $2\pi\,(r+1) - 2\pi r = 2\pi$ extra feet of ribbon.

8. The north pole is such a point. And there are others: Consider a point a_1 near the south pole such that the parallel passing through a_1 forms a circle C_1 with circumference exactly one mile. Any point P_1 exactly one mile north of the circle C_1 along a meridian is a point satisfying the conditions in the problem: starting at P_1 she walks one mile south to the point a_1 on the circle C_1, then one mile east along C_1 returning to the point a_1, then north for one mile to P_1. That's not all. If a point a_2 (or a_3, a_4, a_5, ...) is chosen near the south pole so that the parallel passing through it forms a circle C_2 (C_3, C_4, C_5, ...) with a circumference of exactly $\frac{1}{2}$ mile ($\frac{1}{3}$ mi, $\frac{1}{4}$ mi, $\frac{1}{5}$ mi, ...), then the point P_2 (P_3, P_4, P_5, ...) one mile north of a_2 (a_3, a_4, a_5, ...) along a meridian satisfies the conditions of the problem: she walks one mile south from P_2 (P_3, P_4, P_5, ...) arriving at a_2 (a_3, a_4, a_5, ...) along the circle C_2 (C_3, C_4, C_5, ...), walks east along the circle for one mile thus traversing the circle twice (three times, four times, five times, ...) returning to a_2 (a_3, a_4, a_5, ...), and then walks north one mile to P_2 (P_3, P_4, P_5, ...).

1 FUNDAMENTALS

1.1 REAL NUMBERS

1. **(a)** The natural numbers are $\{1, 2, 3, \ldots\}$.

(b) The numbers $\{\ldots, -3, -2, -1, 0\}$ are integers but not natural numbers.

(c) Any irreducible fraction $\dfrac{p}{q}$ with $q \neq 1$ is rational but is not an integer. Examples: $\frac{3}{2}$, $-\frac{5}{12}$, $\frac{1729}{23}$.

(d) Any number which cannot be expressed as a ratio $\dfrac{p}{q}$ of two integers is irrational. Examples are $\sqrt{2}$, $\sqrt{3}$, π, and e.

2. **(a)** $ab = ba$; Commutative Property of Multiplication

(b) $a + (b + c) = (a + b) + c$; Associative Property of Addition

(c) $a\,(b + c) = ab + ac$; Distributive Property

3. The set of numbers between but not including 2 and 7 can be written as **(a)** $\{x \mid 2 < x < 7\}$ in interval notation, or **(b)** $(2, 7)$ in interval notation.

4. The symbol $|x|$ stands for the *absolute value* of the number x. If x is not 0, then the sign of $|x|$ is always *positive*.

5. The distance between a and b on the real line is $d\,(a, b) = |b - a|$. So the distance between -5 and 2 is $|2 - (-5)| = 7$.

6. **(a)** Yes, the sum of two rational numbers is rational: $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad + bc}{bd}$.

(b) No, the sum of two irrational numbers can be irrational ($\pi + \pi = 2\pi$) or rational ($-\pi + \pi = 0$).

7. **(a)** No: $a - b = -(b - a) \neq b - a$ in general.

(b) No; by the Distributive Property, $-2\,(a - 5) = -2a + -2\,(-5) = -2a + 10 \neq -2a - 10$.

8. **(a)** Yes, absolute values (such as the distance between two different numbers) are always positive.

(b) Yes, $|b - a| = |a - b|$.

9. **(a)** Natural number: 100

(b) Integers: 0, 100, -8

(c) Rational numbers: -1.5, 0, $\frac{5}{2}$, 2.71, $3.1\overline{4}$, 100, -8

(d) Irrational numbers: $\sqrt{7}$, $-\pi$

10. **(a)** Natural number: $\sqrt{16}\,(= 4)$

(b) Integers: -500, $\sqrt{16}$, $-\frac{20}{5}$ $(= -4)$

(c) Rational numbers: 1.3, $1.3333\ldots$, 5.34, -500, $1\frac{2}{3}$, $\sqrt{16}$, $\frac{246}{579}$, $-\frac{20}{5}$

(d) Irrational number: $\sqrt{5}$

11. Commutative Property of addition

12. Commutative Property of multiplication

13. Associative Property of addition

14. Distributive Property

15. Distributive Property

16. Distributive Property

17. Commutative Property of multiplication

18. Distributive Property

19. $x + 3 = 3 + x$

20. $7\,(3x) = (7 \cdot 3)\,x$

21. $4\,(A + B) = 4A + 4B$

22. $5x + 5y = 5\,(x + y)$

23. $3\,(x + y) = 3x + 3y$

24. $(a - b)\,8 = 8a - 8b$

25. $4\,(2m) = (4 \cdot 2)\,m = 8m$

26. $\frac{4}{3}\,(-6y) = \left[\frac{4}{3}\,(-6)\right] y = -8y$

27. $-\frac{5}{2}(2x-4y) = -\frac{5}{2}(2x) + \frac{5}{2}(4y) = -5x + 10y$

28. $(3a)(b+c-2d) = 3ab + 3ac - 6ad$

29. (a) $\frac{3}{10} + \frac{4}{15} = \frac{9}{30} + \frac{8}{30} = \frac{17}{30}$

(b) $\frac{1}{4} + \frac{1}{5} = \frac{5}{20} + \frac{4}{20} = \frac{9}{20}$

30. (a) $\frac{2}{3} - \frac{3}{5} = \frac{10}{15} - \frac{9}{15} = \frac{1}{15}$

(b) $1 + \frac{5}{8} - \frac{1}{6} = \frac{24}{24} + \frac{15}{24} - \frac{4}{24} = \frac{35}{24}$

31. (a) $\frac{2}{3}\left(6 - \frac{3}{2}\right) = \frac{2}{3} \cdot 6 - \frac{2}{3} \cdot \frac{3}{2} = 4 - 1 = 3$

(b) $\left(3 + \frac{1}{4}\right)\left(1 - \frac{4}{5}\right) = \left(\frac{12}{4} + \frac{1}{4}\right)\left(\frac{5}{5} - \frac{4}{5}\right) = \frac{13}{4} \cdot \frac{1}{5} = \frac{13}{20}$

32. (a) $\dfrac{2}{\frac{2}{3}} - \dfrac{\frac{2}{3}}{2} = 2 \cdot \frac{3}{2} - \frac{2}{3} \cdot \frac{1}{2} = 3 - \frac{1}{3} = \frac{9}{3} - \frac{1}{3} = \frac{8}{3}$

(b) $\dfrac{\frac{2}{5} + \frac{1}{2}}{\frac{1}{10} + \frac{3}{15}} = \dfrac{\frac{2}{5} + \frac{1}{2}}{\frac{1}{10} + \frac{1}{5}} = \dfrac{\frac{2}{5} + \frac{1}{2}}{\frac{1}{10} + \frac{1}{5}} \cdot \frac{10}{10} = \frac{4+5}{1+2} = \frac{9}{3} = 3$

33. (a) $2 \cdot 3 = 6$ and $2 \cdot \frac{7}{2} = 7$, so $3 < \frac{7}{2}$

(b) $-6 > -7$

(c) $3.5 = \frac{7}{2}$

34. (a) $3 \cdot \frac{2}{3} = 2$ and $3 \cdot 0.67 = 2.01$, so $\frac{2}{3} < 0.67$

(b) $\frac{2}{3} > -0.67$

(c) $|0.67| = |-0.67|$

35. (a) False

(b) True

36. (a) False: $\sqrt{3} \approx 1.73205 < 1.7325$.

(b) False

37. (a) True **(b)** False

38. (a) True **(b)** True

39. (a) $x > 0$ **(b)** $t < 4$

(c) $a \geq \pi$ **(d)** $-5 < x < \frac{1}{3}$

(e) $|3 - p| \leq 5$

40. (a) $y < 0$ **(b)** $z > 1$

(c) $b \leq 8$ **(d)** $0 < w \leq 17$

(e) $|y - \pi| \geq 2$

41. (a) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(b) $A \cap B = \{2, 4, 6\}$

42. (a) $B \cup C = \{2, 4, 6, 7, 8, 9, 10\}$

(b) $B \cap C = \{8\}$

43. (a) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(b) $A \cap C = \{7\}$

44. (a) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(b) $A \cap B \cap C = \varnothing$

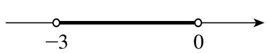
45. (a) $B \cup C = \{x \mid x \leq 5\}$

(b) $B \cap C = \{x \mid -1 < x < 4\}$

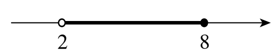
46. (a) $A \cap C = \{x \mid -1 < x \leq 5\}$

(b) $A \cap B = \{x \mid -2 \leq x < 4\}$

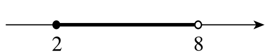
47. $(-3, 0) = \{x \mid -3 < x < 0\}$



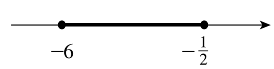
48. $(2, 8] = \{x \mid 2 < x \leq 8\}$



49. $[2, 8) = \{x \mid 2 \leq x < 8\}$



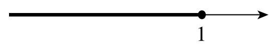
50. $\left[-6, -\frac{1}{2}\right] = \left\{x \mid -6 \leq x \leq -\frac{1}{2}\right\}$



51. $[2, \infty) = \{x \mid x \geq 2\}$

52. $(-\infty, 1) = \{x \mid x < 1\}$

53. $x \le 1 \Leftrightarrow x \in (-\infty, 1]$



54. $1 \le x \le 2 \Leftrightarrow x \in [1, 2]$

55. $-2 < x \le 1 \Leftrightarrow x \in (-2, 1]$

56. $x \ge -5 \Leftrightarrow x \in [-5, \infty)$

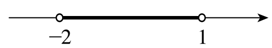
57. $x > -1 \Leftrightarrow x \in (-1, \infty)$

58. $-5 < x < 2 \Leftrightarrow x \in (-5, 2)$

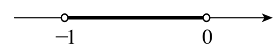
59. **(a)** $[-3, 5]$ **(b)** $(-3, 5]$

60. **(a)** $[0, 2)$ **(b)** $(-2, 0]$

61. $(-2, 0) \cup (-1, 1) = (-2, 1)$



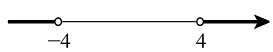
62. $(-2, 0) \cap (-1,) = (-1, 0)$



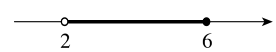
63. $[-4, 6] \cap [0, 8) = [0, 6]$

64. $[-4, 6] \cup [0, 8) = [-4, 8)$

65. $(-\infty, -4) \cup (4, \infty)$



66. $(-\infty, 6] \cap (2, 10) = (2, 6]$



67. **(a)** $|100| = 100$

(b) $|-73| = 73$

68. **(a)** $\left|\sqrt{5} - 5\right| = -\left(\sqrt{5} - 5\right) = 5 - \sqrt{5}$, since $5 > \sqrt{5}$.

(b) $|10 - \pi| = 10 - \pi$, since $10 > \pi$.

69. **(a)** $||-6| - |-4|| = |6 - 4| = |2| = 2$

(b) $\frac{-1}{|-1|} = \frac{-1}{1} = -1$

70. **(a)** $|2 - |-12|| = |2 - 12| = |-10| = 10$

(b) $-1 - |1 - |-1|| = -1 - |1 - 1| = -1 - |0| = -1$

71. **(a)** $|(-2) \cdot 6| = |-12| = 12$

(b) $\left|\left(-\frac{1}{3}\right)(-15)\right| = |5| = 5$

72. **(a)** $\left|\frac{-6}{24}\right| = \left|-\frac{1}{4}\right| = \frac{1}{4}$

(b) $\left|\frac{7-12}{12-7}\right| = \left|-\frac{5}{5}\right| = |-1| = 1$

73. $|(-2) - 3| = |-5| = 5$

74. $|-2.5 - 1.5| = |-4| = 4$

75. **(a)** $|17 - 2| = 15$

(b) $|21 - (-3)| = |21 + 3| = |24| = 24$

(c) $\left|-\frac{3}{10} - \frac{11}{8}\right| = \left|-\frac{12}{40} - \frac{55}{40}\right| = \left|-\frac{67}{40}\right| = \frac{67}{40}$

76. **(a)** $\left|\frac{7}{15} - \left(-\frac{1}{21}\right)\right| = \left|\frac{49}{105} + \frac{5}{105}\right| = \left|\frac{54}{105}\right| = \left|\frac{18}{35}\right| = \frac{18}{35}$

(b) $|-38 - (-57)| = |-38 + 57| = |19| = 19$.

(c) $|-2.6 - (-1.8)| = |-2.6 + 1.8| = |-0.8| = 0.8$.

77. **(a)** Let $x = 0.777\ldots$. So $10x = 7.7777\ldots \Leftrightarrow x = 0.7777\ldots \Leftrightarrow 9x = 7$. Thus, $x = \frac{7}{9}$.

(b) Let $x = 0.2888\ldots$. So $100x = 28.8888\ldots \Leftrightarrow 10x = 2.8888\ldots \Leftrightarrow 90x = 26$. Thus, $x = \frac{26}{90} = \frac{13}{45}$.

(c) Let $x = 0.575757\ldots$. So $100x = 57.5757\ldots \Leftrightarrow x = 0.5757\ldots \Leftrightarrow 99x = 57$. Thus, $x = \frac{57}{99} = \frac{19}{33}$.

78. **(a)** Let $x = 5.2323\ldots$. So $100x = 523.2323\ldots \Leftrightarrow 1x = 5.2323\ldots \Leftrightarrow 99x = 518$. Thus, $x = \frac{518}{99}$.

(b) Let $x = 1.3777\ldots$. So $100x = 137.7777\ldots \Leftrightarrow 10x = 13.7777\ldots \Leftrightarrow 90x = 124$. Thus, $x = \frac{124}{90} = \frac{62}{45}$.

(c) Let $x = 2.13535\ldots$. So $1000x = 2135.3535\ldots \Leftrightarrow 10x = 21.3535\ldots \Leftrightarrow 990x = 2114$. Thus, $x = \frac{2114}{990} = \frac{1057}{495}$.

79. $\pi > 3$, so $|\pi - 3| = \pi - 3$.

80. $\sqrt{2} > 1$, so $\left|1 - \sqrt{2}\right| = \sqrt{2} - 1$.

81. $a < b$, so $|a - b| = -(a - b) = b - a$.

82. $a + b + |a - b| = a + b + b - a = 2b$

83. **(a)** $-a$ is negative because a is positive.

(b) bc is positive because the product of two negative numbers is positive.

(c) $a - ba + (-b)$ is positive because it is the sum of two positive numbers.

(d) $ab + ac$ is negative: each summand is the product of a positive number and a negative number, and the sum of two negative numbers is negative.

84. **(a)** $-b$ is positive because b is negative.

(b) $a + bc$ is positive because it is the sum of two positive numbers.

(c) $c - a = c + (-a)$ is negative because c and $-a$ are both negative.

(d) ab^2 is positive because both a and b^2 are positive.

85. Distributive Property

86.

Day	T_O	T_G	$T_O - T_G$	$\lvert T_O - T_G \rvert$
Sunday	68	77	-9	9
Monday	72	75	-3	3
Tuesday	74	74	0	0
Wednesday	80	75	5	5
Thursday	77	69	8	8
Friday	71	70	1	1
Saturday	70	71	-1	1

$T_O - T_G$ gives more information because it tells us which city had the higher temperature.

87. **(a)** When $L = 60$, $x = 8$, and $y = 6$, we have $L + 2(x + y) = 60 + 2(8 + 6) = 60 + 28 = 88$. Because $88 \le 108$ the post office will accept this package.

When $L = 48$, $x = 24$, and $y = 24$, we have $L + 2(x + y) = 48 + 2(24 + 24) = 48 + 96 = 144$, and since $144 \not\le 108$, the post office will *not* accept this package.

(b) If $x = y = 9$, then $L + 2(9 + 9) \le 108 \Leftrightarrow L + 36 \le 108 \Leftrightarrow L \le 72$. So the length can be as long as 72 in. = 6 ft.

88. Let $x = \dfrac{m_1}{n_1}$ and $y = \dfrac{m_2}{n_2}$ be rational numbers. Then $x + y = \dfrac{m_1}{n_1} + \dfrac{m_2}{n_2} = \dfrac{m_1 n_2 + m_2 n_1}{n_1 n_2}$, $x - y = \dfrac{m_1}{n_1} - \dfrac{m_2}{n_2} = \dfrac{m_1 n_2 - m_2 n_1}{n_1 n_2}$, and $x \cdot y = \dfrac{m_1}{n_1} \cdot \dfrac{m_2}{n_2} = \dfrac{m_1 m_2}{n_1 n_2}$. This shows that the sum, difference, and product of two rational numbers are again rational numbers. However the product of two irrational numbers is not necessarily irrational; for example, $\sqrt{2} \cdot \sqrt{2} = 2$, which is rational. Also, the sum of two irrational numbers is not necessarily irrational; for example, $\sqrt{2} + \left(-\sqrt{2}\right) = 0$ which is rational.

89. $\frac{1}{2}+\sqrt{2}$ is irrational. If it were rational, then by Exercise 6(a), the sum $\left(\frac{1}{2}+\sqrt{2}\right)+\left(-\frac{1}{2}\right)=\sqrt{2}$ would be rational, but this is not the case.

Similarly, $\frac{1}{2}\cdot\sqrt{2}$ is irrational.

(a) Following the hint, suppose that $r+t=q$, a rational number. Then by Exercise 6(a), the sum of the two rational numbers $r+t$ and $-r$ is rational. But $(r+t)+(-r)=t$, which we know to be irrational. This is a contradiction, and hence our original premise—that $r+t$ is rational—was false.

(b) r is rational, so $r=\dfrac{a}{b}$ for some integers a and b. Let us assume that $rt=q$, a rational number. Then by definition, $q=\dfrac{c}{d}$ for some integers c and d. But then $rt=q\Leftrightarrow\dfrac{a}{b}t=\dfrac{c}{d}$, whence $t=\dfrac{bc}{ad}$, implying that t is rational. Once again we have arrived at a contradiction, and we conclude that the product of a rational number and an irrational number is irrational.

90.

x	1	2	10	100	1000
$\frac{1}{x}$	1	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

As x gets large, the fraction $1/x$ gets small. Mathematically, we say that $1/x$ goes to zero.

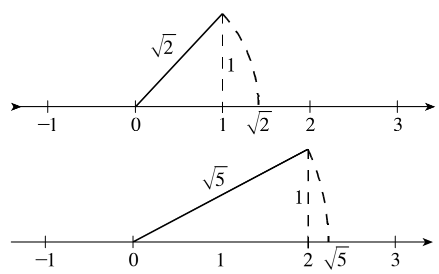
x	1	0.5	0.1	0.01	0.001
$\frac{1}{x}$	1	$\frac{1}{0.5}=2$	$\frac{1}{0.1}=10$	$\frac{1}{0.01}=100$	$\frac{1}{0.001}=1000$

As x gets small, the fraction $1/x$ gets large. Mathematically, we say that $1/x$ goes to infinity.

91. We can construct the number $\sqrt{2}$ on the number line by transferring the length of the hypotenuse of a right triangle with legs of length 1 and 1.

Similarly, to locate $\sqrt{5}$, we construct a right triangle with legs of length 1 and 2. By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{1^2+2^2}=\sqrt{5}$. Then transfer the length of the hypotenuse to the number line.

The square root of any rational number can be located on a number line in this fashion.



The circle in the second figure in the text has circumference π, so if we roll it along a number line one full rotation, we have found π on the number line. Similarly, any rational multiple of π can be found this way.

92. (a) Suppose that $a>b$, so $\max(a,b)=a$ and $|a-b|=a-b$. Then $\dfrac{a+b+|a-b|}{2}=\dfrac{a+b+a-b}{2}=a$.

On the other hand, if $b>a$, then $\max(a,b)=b$ and $|a-b|=-(a-b)=b-a$. In this case, $\dfrac{a+b+|a-b|}{2}=\dfrac{a+b+b-a}{2}=b$.

If $a=b$, then $|a-b|=0$ and the result is trivial.

(b) If $a<b$, then $\min(a,b)=a$ and $|a-b|=b-a$. In this case $\dfrac{a+b-|a-b|}{2}=\dfrac{a+b-(b-a)}{2}=a$.

Similarly, if $b<a$, then $\dfrac{a+b-|a-b|}{2}=b$; and if $a=b$, the result is trivial.

93. Answers will vary.

94. (a) Subtraction is not commutative. For example, $5-1\neq 1-5$.

(b) Division is not commutative. For example, $5\div 1\neq 1\div 5$.

(c) Putting on your socks and putting on your shoes are not commutative. If you put on your socks first, then your shoes, the result is not the same as if you proceed the other way around.

(d) Putting on your hat and putting on your coat are commutative. They can be done in either order, with the same result.

(e) Washing laundry and drying it are not commutative.

95. **(a)** If $x = 2$ and $y = 3$, then $|x + y| = |2 + 3| = |5| = 5$ and $|x| + |y| = |2| + |3| = 5$.
If $x = -2$ and $y = -3$, then $|x + y| = |-5| = 5$ and $|x| + |y| = 5$.
If $x = -2$ and $y = 3$, then $|x + y| = |-2 + 3| = 1$ and $|x| + |y| = 5$.
In each case, $|x + y| \le |x| + |y|$ and the Triangle Inequality is satisfied.

(b) *Case 0:* If either x or y is 0, the result is equality, trivially.

Case 1: If x and y have the same sign, then $|x + y| = \left\{ \begin{array}{ll} x + y & \text{if } x \text{ and } y \text{ are positive} \\ -(x + y) & \text{if } x \text{ and } y \text{ are negative} \end{array} \right\} = |x| + |y|$.

Case 2: If x and y have opposite signs, then suppose without loss of generality that $x < 0$ and $y > 0$. Then $|x + y| < |-x + y| = |x| + |y|$.

1.2 EXPONENTS AND RADICALS

1. **(a)** Using exponential notation we can write the product $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ as 5^6.

(b) In the expression 3^4, the number 3 is called the *base* and the number 4 is called the *exponent*.

2. **(a)** When we multiply two powers with the same base, we *add* the exponents. So $3^4 \cdot 3^5 = 3^9$.

(b) When we divide two powers with the same base, we *subtract* the exponents. So $\dfrac{3^5}{3^2} = 3^3$.

3. **(a)** Using exponential notation we can write $\sqrt[3]{5}$ as $5^{1/3}$.

(b) Using radicals we can write $5^{1/2}$ as $\sqrt{5}$.

(c) No. $\sqrt{5^2} = \left(5^2\right)^{1/2} = 5^{2(1/2)} = 5$ and $\left(\sqrt{5}\right)^2 = \left(5^{1/2}\right)^2 = 5^{(1/2)2} = 5$.

4. $\left(4^{1/2}\right)^3 = 2^3 = 8$; $\left(4^3\right)^{1/2} = 64^{1/2} = 8$

5. Because the denominator is of the form $\sqrt{a}$, we multiply numerator and denominator by $\sqrt{a}$: $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

6. $5^{1/3} \cdot 5^{2/3} = 5^1 = 5$

7. **(a)** No, $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$.

(b) Yes, $(-5)^4 = 625$, while $-5^4 = -\left(5^4\right) = -625$.

8. **(a)** No, $\left(x^2\right)^3 = x^{2 \cdot 3} = x^6$.

(b) No, $\left(2x^4\right)^3 = 2^3 x^{4 \cdot 3} = 8x^{12}$.

(c) No; if a is negative, then $\sqrt{4a^2} = -2a$.

(d) No, because $(a + 2)^2 \ne a^2 + 4$.

9. $\dfrac{1}{\sqrt{3}} = 3^{-1/2}$

10. $\sqrt[3]{7^2} = 7^{2/3}$

11. $4^{2/3} = \sqrt[3]{4^2} = \sqrt[3]{16}$

12. $10^{-3/2} = \left(10^{3/2}\right)^{-1} = \left(\sqrt{10^3}\right)^{-1} = \dfrac{1}{\sqrt{10^3}}$

13. $\sqrt[5]{5^3} = 5^{3/5}$

14. $2^{-1.5} = 2^{-3/2} = \dfrac{1}{\sqrt{2^3}} = \dfrac{1}{\sqrt{8}}$

15. $a^{2/5} = \sqrt[5]{a^2}$

16. $\dfrac{1}{\sqrt{x^5}} = \dfrac{1}{x^{5/2}} = x^{-5/2}$

17. **(a)** $-2^6 = -\left(2^6\right) = -64$

(b) $(-2)^6 = 64$

(c) $\left(\frac{1}{5}\right)^2 \cdot (-3)^3 = \dfrac{(-3)^3}{5^2} = -\dfrac{27}{25}$

18. **(a)** $(-5)^3 = -125$

(b) $-5^3 = -\left(5^3\right) = -125$

(c) $(-5)^2 \cdot \left(\frac{2}{5}\right)^2 = 4$

19. **(a)** $\left(\dfrac{5}{3}\right)^0 \cdot 2^{-1} = \dfrac{1}{2}$

(b) $\dfrac{2^{-3}}{3^0} = \dfrac{1}{2^3} = \dfrac{1}{8}$

(c) $\left(\dfrac{2}{3}\right)^{-2} = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$

20. **(a)** $-2^3 \cdot (-2)^0 = -\left(2^3\right) \cdot 1 = -8$

(b) $-2^{-3} \cdot (-2)^0 = -\dfrac{1}{2^3} \cdot 1 = -\dfrac{1}{8}$

(c) $\left(\dfrac{-3}{5}\right)^{-3} = \dfrac{5^3}{(-3)^3} = -\dfrac{125}{27}$

21. **(a)** $5^3 \cdot 5 = 5^{3+1} = 625$

(b) $5^4 \cdot 5^{-2} = 5^{4-2} = 25$

(c) $\left(2^2\right)^3 = 2^{2 \cdot 3} = 64$

22. **(a)** $3^8 \cdot 3^5 = 3^{8+5} = 3^{13}$

(b) $\dfrac{10^7}{10^4} = 10^{7-4} = 1000$

(c) $\left(3^5\right)^4 = 3^{5 \cdot 4} = 3^{20}$

23. **(a)** $3\sqrt[3]{16} = 3\sqrt[3]{8 \cdot 2} = 3\sqrt[3]{8}\sqrt[3]{2} = 6\sqrt[3]{2}$

(b) $\dfrac{\sqrt{18}}{\sqrt{81}} = \sqrt{\dfrac{18}{81}} = \sqrt{\dfrac{2}{9}} = \dfrac{\sqrt{2}}{3}$

(c) $\sqrt{\dfrac{27}{4}} = \dfrac{3\sqrt{3}}{2} = \dfrac{\sqrt{9 \cdot 3}}{\sqrt{4}}$

24. **(a)** $2\sqrt[3]{81} = 2\sqrt[3]{27 \cdot 3} = 2\sqrt[3]{27}\sqrt[3]{3} = 6\sqrt[3]{3}$

(b) $\dfrac{\sqrt{18}}{\sqrt{25}} = \dfrac{\sqrt{9 \cdot 2}}{5} = \dfrac{3\sqrt{2}}{5}$

(c) $\sqrt{\dfrac{12}{49}} = \dfrac{\sqrt{4 \cdot 3}}{\sqrt{49}} = \dfrac{2\sqrt{3}}{7}$

25. **(a)** $\sqrt{3}\sqrt{15} = \sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$

(b) $\dfrac{\sqrt{48}}{\sqrt{3}} = \sqrt{\dfrac{48}{3}} = \sqrt{16} = 4$

(c) $\sqrt[3]{24}\sqrt[3]{18} = \sqrt[3]{24 \cdot 18} = \sqrt[3]{6^3 \cdot 2} = 6\sqrt[3]{2}$

26. **(a)** $\sqrt{10}\sqrt{32} = \sqrt{320} = \sqrt{64 \cdot 5} = 8\sqrt{5}$

(b) $\dfrac{\sqrt{54}}{\sqrt{6}} = \sqrt{\dfrac{54}{6}} = 3$

(c) $\sqrt[3]{15}\sqrt[3]{75} = \sqrt[3]{15 \cdot 75} = \sqrt[3]{5^3 \cdot 9} = 5\sqrt[3]{9}$

27. **(a)** $\dfrac{\sqrt{132}}{\sqrt{3}} = \sqrt{\dfrac{132}{3}} = \sqrt{44} = \sqrt{4 \cdot 11} = 2\sqrt{11}$

(b) $\sqrt[3]{2}\sqrt[3]{32} = \sqrt[3]{64} = 4$

(c) $\sqrt[4]{\dfrac{1}{4}}\sqrt[4]{\dfrac{1}{64}} = \sqrt[4]{\dfrac{1}{4 \cdot 4^3}} = \dfrac{1}{4}$

28. **(a)** $\sqrt[5]{\dfrac{1}{8}}\sqrt[5]{\dfrac{1}{4}} = \sqrt[5]{\dfrac{1}{2^3 \cdot 2^2}} = \dfrac{1}{2}$

(b) $\sqrt[6]{\dfrac{1}{2}}\sqrt[6]{128} = \sqrt[6]{\dfrac{2^7}{2}} = 2$

(c) $\dfrac{\sqrt[3]{4}}{\sqrt[3]{108}} = \sqrt[3]{\dfrac{4}{108}} = \sqrt[3]{\dfrac{1}{27}} = \dfrac{1}{3}$

29. **(a)** $x^3 \cdot x^4 = x^{3+4} = x^7$

(b) $\left(2y^2\right)^3 = 2^3 y^{2 \cdot 3} = 8y^6$

(c) $y^{-2}y^7 = y^{-2+7} = y^5$

30. **(a)** $y^5 \cdot y^2 = y^{5+2} = y^7$

(b) $(8x)^2 = 8^2 x^2 = 64x^2$

(c) $x^4 x^{-3} = x^{4-3} = x^1 = x$

31. **(a)** $x^{-5} \cdot x^3 = x^{-5+3} = x^{-2} = \dfrac{1}{x^2}$

(b) $w^{-2}w^{-4}w^5 = w^{-2-4+5} = w^{-1} = \dfrac{1}{w}$

(c) $\dfrac{x^{16}}{x^{10}} = x^{16-10} = x^6$

32. **(a)** $y^2 \cdot y^{-5} = y^{2-5} = y^{-3} = \dfrac{1}{y^3}$

(b) $z^5 z^{-3} z^{-4} = z^{5-3-4} = z^{-2} = \dfrac{1}{z^2}$

(c) $\dfrac{y^7 y^0}{y^{10}} = y^{7+0-10} = y^{-3} = \dfrac{1}{y^3}$

33. (a) $\dfrac{a^9a^{-2}}{a} = a^{9-2-1} = a^6$

(b) $\left(a^2a^4\right)^3 = \left(a^{2+4}\right)^3 = \left(a^6\right)^3 = a^{6\cdot 3} = a^{18}$

(c) $\left(\dfrac{x}{2}\right)^3\left(5x^6\right) = \dfrac{x^3 \cdot 5x^6}{2^3} = \dfrac{5x^9}{8}$

34. (a) $\dfrac{z^2z^4}{z^3z^{-1}} = z^{2+4-3-(-1)} = z^4$

(b) $\left(2a^3a^2\right)^4 = 2^4\left(a^{3+2}\right)^4 = 16\left(a^5\right)^4 = 16a^{20}$

(c) $\left(-3z^2\right)^3\left(2z^3\right) = (-3)^3 \cdot z^{2\cdot 3} \cdot 2z^3 = -54z^9$

35. (a) $\left(3x^3y^2\right)\left(2y^3\right) = 3 \cdot 2x^3y^2y^3 = 6x^3y^5$

(b) $\left(5w^2z^{-2}\right)^2\left(z^3\right) = \left(\dfrac{5w^2}{z^2}\right)^2\left(z^3\right) = \dfrac{5^2\left(w^2\right)^2z^3}{\left(z^2\right)^2} = \dfrac{25w^4}{z}$

36. (a) $\left(8m^{-2}n^4\right)\left(\frac{1}{2}n^{-2}\right) = 8 \cdot \frac{1}{2}m^{-2}n^{4-2} = \dfrac{4n^2}{m^2}$

(b) $\left(3a^4b^{-2}\right)^3\left(a^2b^{-1}\right) = \left(3^3a^{4\cdot 3}b^{-2\cdot 3}\right)\left(a^2b^{-1}\right) = 3^3a^{12+2}b^{-6-1} = \dfrac{27a^{14}}{b^7}$

37. (a) $\dfrac{x^2y^{-1}}{x^{-5}} = x^{2-(-5)}y^{-1} = \dfrac{x^7}{y}$

(b) $\left(\dfrac{a^3}{2b^2}\right)^3 = \dfrac{a^{3\cdot 3}}{2^3\left(b^2\right)^3} = \dfrac{a^9}{8b^6}$

38. (a) $\dfrac{y^{-2}z^{-3}}{y^{-1}} = \dfrac{y}{y^2z^3} = \dfrac{1}{yz^3}$

(b) $\left(\dfrac{x^3y^{-2}}{x^{-3}y^2}\right)^{-2} = \left[x^{3-(-3)}y^{-2-2}\right]^{-2} = \left(x^6y^{-4}\right)^{-2} = \dfrac{y^8}{x^{12}}$

39. (a) $\left(\dfrac{a^2}{b}\right)^5\left(\dfrac{a^3b^2}{c^3}\right)^3 = a^{2\cdot 5+3\cdot 3}b^{-5+2\cdot 3}c^{-3\cdot 3} = \dfrac{a^{19}b}{c^9}$

(b) $\dfrac{\left(u^{-1}v^2\right)^2}{\left(u^3v^{-2}\right)^3} = u^{-1\cdot 2-3\cdot 3}v^{2\cdot 2-(-2)3} = \dfrac{v^{10}}{u^{11}}$

40. (a) $\left(\dfrac{x^4z^2}{4y^5}\right)\left(\dfrac{2x^3y^2}{z^3}\right)^2 = \dfrac{2^2}{4}x^{4+3\cdot 2}y^{-5+2\cdot 2}z^{2+(-3)2} = \dfrac{x^{10}}{yz^4}$

(b) $\dfrac{\left(rs^2\right)^3}{\left(r^{-3}s^2\right)^2} = r^{3-(-3)2}s^{2\cdot 3-2\cdot 2} = r^9s^2$

41. (a) $\dfrac{8a^3b^{-4}}{2a^{-5}b^5} = 4a^{3-(-5)}b^{-4-5} = \dfrac{4a^8}{b^9}$

(b) $\left(\dfrac{y}{5x^{-2}}\right)^{-3} = 5^{-1(-3)}x^{-(-2)(-3)}y^{-3} = \dfrac{125}{x^6y^3}$

42. (a) $\dfrac{5xy^{-2}}{x^{-1}y^{-3}} = 5x^{1-(-1)}y^{-2-(-3)} = 5x^2y$

(b) $\left(\dfrac{2a^{-1}b}{a^2b^{-3}}\right)^{-3} = 2^{-3}a^{-1(-3)-2(-3)}b^{-3-(-3)(-3)} = \dfrac{a^9}{8b^{12}}$

43. (a) $\left(\dfrac{3a}{b^3}\right)^{-1} = 3^{-1}a^{-1}b^{-3(-1)} = \dfrac{b^3}{3a}$

(b) $\left(\dfrac{q^{-1}r^{-1}s^{-2}}{r^{-5}sq^{-8}}\right)^{-1} = \dfrac{r^{-5}sq^{-8}}{q^{-1}r^{-1}s^{-2}} = q^{-8-(-1)}r^{-5-(-1)}s^{1-(-2)} = \dfrac{s^3}{q^7r^4}$

44. **(a)** $\left(\dfrac{s^2t^{-4}}{5s^{-1}t}\right)^{-2} = s^{2(-2)-(-1)(-2)}t^{-4(-2)-1(-2)}5^{-(-2)} = \dfrac{25t^{10}}{s^6}$

(b) $\left(\dfrac{xy^{-2}z^{-3}}{x^2y^3z^{-4}}\right)^{-3} = x^{-3-2(-3)}y^{-2(-3)-3(-3)}z^{-3(-3)-(-4)(-3)} = \dfrac{x^3y^{15}}{z^3}$

45. **(a)** $\sqrt[4]{x^4} = |x|$ **(b)** $\sqrt[4]{16x^8} = \sqrt[4]{2^4x^8} = 2x^2$

46. **(a)** $\sqrt[5]{x^{10}} = \left(x^{10}\right)^{1/5} = x^2$ **(b)** $\sqrt[3]{x^3y^6} = \left(x^3y^6\right)^{1/3} = xy^2$

47. **(a)** $\sqrt[6]{64a^6b^7} = \sqrt[6]{2^6 \cdot a^6 \cdot b^6 \cdot b} = 2ab\sqrt[6]{b}$

(b) $\sqrt[3]{a^2b}\sqrt[3]{64a^4b} = \sqrt[3]{a^2b \cdot 64a^4b} = \sqrt[3]{64a^6b^2} = \sqrt[3]{4^3\left(a^2\right)^3b^2} = 4a^2\sqrt[3]{b^2}$

48. **(a)** $\sqrt[4]{x^4y^2z^2} = \sqrt[4]{x^4}\sqrt[4]{y^2z^2} = |x|\sqrt[4]{y^2z^2} = |x|\sqrt{|yz|}$ **(b)** $\sqrt[3]{\sqrt{64x^6}} = \left(8\left|x^3\right|\right)^{1/3} = 2|x|$

49. **(a)** $\sqrt{32} + \sqrt{18} = \sqrt{16 \cdot 2} + \sqrt{9 \cdot 2} = \sqrt{16}\sqrt{2} + \sqrt{9}\sqrt{2} = 4\sqrt{2} + 3\sqrt{2} = 7\sqrt{2}$

(b) $\sqrt{75} + \sqrt{48} = \sqrt{25 \cdot 3} + \sqrt{16 \cdot 3} = \sqrt{25}\sqrt{3} + \sqrt{16}\sqrt{3} = 5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$

50. **(a)** $\sqrt{125} + \sqrt{45} = \sqrt{25 \cdot 5} + \sqrt{9 \cdot 5} = 5\sqrt{5} + 3\sqrt{3} = 8\sqrt{5}$

(b) $\sqrt[3]{54} - \sqrt[3]{16} = \sqrt[3]{27 \cdot 2} - \sqrt[3]{8 \cdot 2} = \sqrt[3]{27}\sqrt[3]{2} - \sqrt[3]{8}\sqrt[3]{2} = \sqrt[3]{2}$

51. **(a)** $\sqrt{9a^3} + \sqrt{a} = \sqrt{9}\sqrt{a^2 \cdot a} + \sqrt{a} = 3a\sqrt{a} + \sqrt{a} = (3a+1)\sqrt{a}$

(b) $\sqrt{16x} + \sqrt{x^5} = \sqrt{16}\sqrt{x} + \sqrt{x^4}\sqrt{x} = \left(x^2+4\right)\sqrt{x}$

52. **(a)** $\sqrt[3]{x^4} + \sqrt[3]{8x} = \sqrt[3]{x^3 \cdot x} + \sqrt[3]{2^3x} = (x+2)\sqrt[3]{x}$

(b) $4\sqrt{18rt^3} + 5\sqrt{32r^3t^5} = 4\sqrt{\left(3^2 \cdot 2\right)r\left(t^2 \cdot t\right)} + 5\sqrt{\left(4^2 \cdot 2\right)\left(r^2 \cdot r\right)\left(t^4 \cdot t\right)} = 4(3)(t)\sqrt{2rt} + 5(4)(r)\left(t^2\right)\sqrt{2rt}$

$= \left(20rt^2 + 12t\right)\sqrt{2rt}$

53. **(a)** $\sqrt{81x^2+81} = \sqrt{9^2\left(x^2+1\right)} = 9\sqrt{x^2+1}$ **(b)** $\sqrt{36x^2+36y^2} = \sqrt{6^2\left(x^2+y^2\right)} = 6\sqrt{x^2+y^2}$

54. **(a)** $\sqrt{27a^2+63a} = \sqrt{3^2a(3a+7)} = 3\sqrt{a(3a+7)}$ **(b)** $\sqrt{75t+100t^2} = \sqrt{5^2t(3+4t)} = 5\sqrt{t(4t+3)}$

55. **(a)** $16^{1/4} = \sqrt[4]{16} = 2$ **(b)** $-8^{1/3} = \sqrt[3]{-8} = -2$ **(c)** $9^{-1/2} = \dfrac{1}{\sqrt{9}} = \dfrac{1}{3}$

56. **(a)** $27^{1/3} = \sqrt[3]{27} = 3$ **(b)** $(-8)^{1/3} = \sqrt[3]{-8} = -2$ **(c)** $-\left(\dfrac{1}{8}\right)^{1/3} = -\dfrac{1}{\sqrt[3]{8}} = -\dfrac{1}{2}$

57. **(a)** $32^{2/5} = \left(\sqrt[5]{32}\right)^2 = 2^2 = 4$ **(b)** $\left(\dfrac{4}{9}\right)^{-1/2} = \dfrac{1}{\sqrt{4/9}} = \sqrt{\dfrac{9}{4}} = \dfrac{3}{2}$ **(c)** $\left(\dfrac{16}{81}\right)^{3/4} = \left(\dfrac{\sqrt[4]{16}}{\sqrt[4]{81}}\right)^3 = \dfrac{8}{27}$

58. **(a)** $125^{2/3} = \left(\sqrt[3]{125}\right)^2 = 5^2 = 25$

(b) $\left(\dfrac{25}{64}\right)^{3/2} = \left(\dfrac{\sqrt{25}}{\sqrt{64}}\right)^3 = \left(\dfrac{5}{8}\right)^3 = \dfrac{125}{512}$

(c) $27^{-4/3} = \dfrac{1}{\left(\sqrt[3]{27}\right)^4} = \dfrac{1}{3^4} = \dfrac{1}{81}$

59. **(a)** $5^{2/3} \cdot 5^{1/3} = 5^{(2/3)+(1/3)} = 5^1 = 5$

(b) $\dfrac{3^{3/5}}{3^{2/5}} = 3^{(3/5)-(2/5)} = 3^{1/5} = \sqrt[5]{3}$

(c) $\left(\sqrt[3]{4}\right)^3 = \left(4^{1/3}\right)^3 = 4^{(1/3)\cdot 3} = 4^1 = 4$

60. (a) $3^{2/7} \cdot 3^{12/7} = 3^{2/7+12/7} = 3^2 = 9$

(b) $\dfrac{7^{2/3}}{7^{5/3}} = 7^{(2/3)-(5/3)} = 7^{-1} = \dfrac{1}{7}$

(c) $\left(\sqrt[5]{6}\right)^{-10} = \left(6^{1/5}\right)^{-10} = 6^{(1/5)(-10)} = 6^{-2} = \dfrac{1}{6^2} = \dfrac{1}{36}$

61. (a) $x^{3/4}x^{5/4} = x^{3/4+5/4} = x^2$

(b) $y^{2/3}y^{4/3} = y^{2/3+4/3} = y^2$

62. (a) $(4b)^{1/2}\left(8b^{1/4}\right) = 4^{1/2} \cdot 8b^{1/2+1/4} = 16b^{3/4}$

(b) $\left(3a^{3/4}\right)^2\left(5a^{1/2}\right) = 3^2 \cdot 5a^{(3/4)\cdot 2+(1/2)} = 45a^2$

63. (a) $\dfrac{w^{4/3}w^{2/3}}{w^{1/3}} = w^{4/3+2/3-1/3} = w^{5/3}$

(b) $\dfrac{a^{5/4}\left(2a^{3/4}\right)^3}{a^{1/4}} = 2^3a^{(5/4)+(3/4)3-1/4} = 8a^{13/4}$

64. (a) $\left(8y^3\right)^{-2/3} = 8^{-2/3}y^{3(-2/3)} = \dfrac{1}{4y^2}$

(b) $\left(u^4v^6\right)^{-1/3} = u^{4(-1/3)}v^{6(-1/3)} = \dfrac{1}{u^{4/3}v^2}$

65. (a) $\left(8a^6b^{3/2}\right)^{2/3} = 8^{2/3}a^{6(2/3)}b^{(3/2)(2/3)} = 4a^4b$

(b) $\left(4a^6b^8\right)^{3/2} = 4^{3/2}a^{6(3/2)}b^{8(3/2)} = 8a^9b^{12}$

66. (a) $\left(x^{-5}y^{1/3}\right)^{-3/5} = x^{-5(-3/5)}y^{(1/3)(-3/5)} = \dfrac{x^3}{y^{1/5}}$

(b) $\left(4r^8s^{-1/2}\right)^{1/2}\left(32s^{-5/4}\right)^{-1/5} = 4^{1/2}r^{8(1/2)}s^{(-1/2)(1/2)}32^{-1/5}s^{(-5/4)(-1/5)} = 2r^4s^{-1/4} \cdot \frac{1}{2}s^{1/4} = r^4$

67. (a) $\dfrac{\left(8s^3t^3\right)^{2/3}}{\left(s^4t^{-8}\right)^{1/4}} = 8^{2/3}s^{3(2/3)-4(1/4)}t^{3(2/3)-(-8)(1/4)} = 4st^4$

(b) $\dfrac{\left(32x^5y^{-3/2}\right)^{2/5}}{\left(x^{5/3}y^{2/3}\right)^{3/5}} = \dfrac{32^{2/5}x^{5(2/5)}y^{(-3/2)(2/5)}}{x^{(5/3)(3/5)}y^{(2/3)(3/5)}} = \dfrac{4x^2y^{-3/5}}{xy^{2/5}} = 4x^{2-1}y^{(-3/5)-(2/5)} = \dfrac{4x}{y}$

68. (a) $\left(\dfrac{x^8y^{-4}}{16y^{4/3}}\right)^{-1/4} = 16^{-(-1/4)}x^{8(-1/4)}y^{-4(-1/4)-4/3(-1/4)} = \dfrac{2y^{4/3}}{x^2}$

(b) $\left(\dfrac{4s^3t^4}{s^2t^{9/2}}\right)^{-1/2} = \left[4s^{3-2}t^{4-(9/2)}\right]^{-1/2} = \left(4st^{-1/2}\right)^{-1/2} = \dfrac{\sqrt[4]{t}}{2\sqrt{s}}$

69. (a) $\left(\dfrac{x^{3/2}}{y^{-1/2}}\right)^4\left(\dfrac{x^{-2}}{y^3}\right) = x^{(3/2)4}y^{-(-1/2)4}x^{-2}y^{-3} = x^{6-2}y^{2-3} = \dfrac{x^4}{y}$

(b) $\left(\dfrac{4y^3z^{2/3}}{x^{1/2}}\right)^2\left(\dfrac{x^{-3}y^6}{8z^4}\right)^{1/3} = 4^28^{-1/3}x^{-1/2(2)-3(1/3)}y^{3(2)+6(1/3)}z^{2/3(2)-4(1/3)} = \dfrac{8y^8}{x^2}$

70. (a) $\left(\dfrac{a^{1/6}b^{-3}}{x^{-1}y}\right)^3\left(\dfrac{x^{-2}b^{-1}}{a^{3/2}y^{1/3}}\right) = a^{1/6(3)-3/2}b^{-3(3)-1}x^{-(-1)(3)-2}y^{-1(3)-1/3} = \dfrac{x}{ab^{10}y^{10/3}}$

(b) $\dfrac{(9st)^{3/2}}{\left(27s^3t^{-4}\right)^{2/3}}\left(\dfrac{3s^{-2}}{4t^{1/3}}\right)^{-1} = 9^{3/2}27^{-2/3}3^{-1}4^{-(-1)}s^{3/2-3(2/3)-2(-1)}t^{3/2-(-4)(2/3)-1/3(-1)} = 4s^{3/2}t^{9/2}$

71. **(a)** $\sqrt{x^3} = \left(x^3\right)^{1/2} = x^{3(1/2)} = x^{3/2}$

(b) $\sqrt[5]{x^6} = \left(x^6\right)^{1/5} = x^{6(1/5)} = x^{5/6}$

72. **(a)** $\sqrt{x^5} = \left(x^5\right)^{1/2} = x^{5(1/2)} = x^{5/2}$

(b) $\sqrt[4]{x^6} = \left(x^6\right)^{1/4} = x^{6(1/4)} = x^{3/2}$

73. **(a)** $\sqrt[6]{y^5}\sqrt[3]{y^2} = y^{5/6} \cdot y^{2/3} = y^{5/6+2/3} = y^{3/2}$

(b) $\left(5\sqrt[3]{x}\right)\left(2\sqrt[4]{x}\right) = 5 \cdot 2x^{1/3+1/4} = 10x^{7/12}$

74. **(a)** $\sqrt[4]{b^3}\sqrt{b} = b^{3/4+1/2} = b^{5/4}$

(b) $\left(2\sqrt{a}\right)\left(\sqrt[3]{a^2}\right) = 2a^{1/2+2/3} = 2a^{7/6}$

75. **(a)** $\sqrt{4st^3}\sqrt[6]{s^3t^2} = 4^{1/2}s^{1/2+3/6}t^{3/2+2/6} = 2st^{11/6}$

(b) $\dfrac{\sqrt[4]{x^7}}{\sqrt[4]{x^3}} = x^{7/4-3/4} = x$

76. **(a)** $\sqrt[5]{x^3y^2}\sqrt[10]{x^4y^{16}} = x^{3/5+4/10}y^{2/5+16/10} = xy^2$

(b) $\dfrac{\sqrt[3]{8x^2}}{\sqrt{x}} = 8^{1/3}x^{2/3-1/2} = 2x^{1/6}$

77. **(a)** $\sqrt[3]{y\sqrt{y}} = \left(y^{1+1/2}\right)^{1/3} = y^{(3/2)(1/3)} = y^{1/2}$

(b) $\sqrt{\dfrac{16u^3v}{uv^5}} = \sqrt{\dfrac{16u^2}{v^4}} = \dfrac{4u}{v^2}$

78. **(a)** $\sqrt{s\sqrt{s^3}} = \left(s^{1+3/2}\right)^{1/2} = s^{5/4}$

(b) $\sqrt[3]{\dfrac{54x^2y^4}{2x^5y}} = \sqrt[3]{\dfrac{27y^3}{x^3}} = \dfrac{3y}{x}$

79. **(a)** $\dfrac{1}{\sqrt{6}} = \dfrac{1}{\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{\sqrt{6}}{6}$

(b) $\sqrt{\dfrac{3}{2}} = \sqrt{\dfrac{3}{2}}\sqrt{\dfrac{2}{2}} = \dfrac{\sqrt{6}}{2}$

(c) $\dfrac{9}{\sqrt[4]{2}} = \dfrac{9}{2^{1/4}} \cdot \dfrac{2^{3/4}}{2^{3/4}} = \dfrac{9\sqrt[4]{8}}{2}$

80. **(a)** $\dfrac{12}{\sqrt{3}} = \dfrac{12}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{12\sqrt{3}}{3} = 4\sqrt{3}$

(b) $\sqrt{\dfrac{12}{5}} = \sqrt{\dfrac{4 \cdot 3}{5}}\sqrt{\dfrac{5}{5}} = \dfrac{2\sqrt{15}}{5}$

(c) $\dfrac{8}{\sqrt[3]{5^2}} = \dfrac{8}{5^{2/3}} \cdot \dfrac{5^{1/3}}{5^{1/3}} = \dfrac{8\sqrt[3]{5}}{5}$

81. **(a)** $\dfrac{1}{\sqrt{5x}} = \dfrac{1}{\sqrt{5x}} \cdot \dfrac{\sqrt{5x}}{\sqrt{5x}} = \dfrac{\sqrt{5x}}{5x}$

(b) $\sqrt{\dfrac{x}{5}} = \sqrt{\dfrac{x}{5}}\sqrt{\dfrac{5}{5}} = \dfrac{\sqrt{5x}}{5}$

(c) $\sqrt[5]{\dfrac{1}{x^3}} = \dfrac{1}{x^{3/5}} \cdot \dfrac{x^{2/5}}{x^{2/5}} = \dfrac{\sqrt[5]{x^2}}{x}$

82. **(a)** $\sqrt{\dfrac{s}{3t}} = \sqrt{\dfrac{s}{3t}}\sqrt{\dfrac{3t}{3t}} = \dfrac{\sqrt{3st}}{3t}$

(b) $\dfrac{a}{\sqrt[6]{b^2}} = \dfrac{a}{\sqrt[3]{b}} \dfrac{\sqrt[3]{b^2}}{\sqrt[3]{b^2}} = \dfrac{a\sqrt[3]{b^2}}{b}$

(c) $\dfrac{1}{c^{3/5}} = \dfrac{1}{c^{3/5}} \cdot \dfrac{c^{2/5}}{c^{2/5}} = \dfrac{c^{2/5}}{c}$

83. **(a)** $69{,}300{,}000 = 6.93 \times 10^7$

(b) $7{,}200{,}000{,}000{,}000 = 7.2 \times 10^{12}$

(c) $0.000028536 = 2.8536 \times 10^{-5}$

(d) $0.0001213 = 1.213 \times 10^{-4}$

84. **(a)** $129{,}540{,}000 = 1.2954 \times 10^8$

(b) $7{,}259{,}000{,}000 = 7.259 \times 10^9$

(c) $0.0000000014 = 1.4 \times 10^{-9}$

(d) $0.0007029 = 7.029 \times 10^{-4}$

85. **(a)** $3.19 \times 10^5 = 319{,}000$

(b) $2.721 \times 10^8 = 272{,}100{,}000$

(c) $2.670 \times 10^{-8} = 0.00000002670$

(d) $9.999 \times 10^{-9} = 0.000000009999$

86. **(a)** $7.1 \times 10^{14} = 710{,}000{,}000{,}000{,}000$

(b) $6 \times 10^{12} = 6{,}000{,}000{,}000{,}000$

(c) $8.55 \times 10^{-3} = 0.00855$

(d) $6.257 \times 10^{-10} = 0.0000000006257$

87. **(a)** 5,900,000,000,000 mi$= 5.9 \times 10^{12}$ mi

(b) 0.0000000000004 cm $= 4 \times 10^{-13}$ cm

(c) 33 billion billion molecules $= 33 \times 10^9 \times 10^9 = 3.3 \times 10^{19}$ molecules

88. **(a)** 93,000,000 mi $= 9.3 \times 10^7$ mi

(b) 0.000000000000000000000053 g $= 5.3 \times 10^{-23}$ g

(c) 5,970,000,000,000,000,000,000,000 kg $= 5.97 \times 10^{24}$ kg

89. $\left(7.2 \times 10^{-9}\right)\left(1.806 \times 10^{-12}\right) = 7.2 \times 1.806 \times 10^{-9} \times 10^{-12} \approx 13.0 \times 10^{-21} = 1.3 \times 10^{-20}$

90. $\left(1.062 \times 10^{24}\right)\left(8.61 \times 10^{19}\right) = 1.062 \times 8.61 \times 10^{24} \times 10^{19} \approx 9.14 \times 10^{43}$

91. $\dfrac{1.295643 \times 10^{9}}{\left(3.610 \times 10^{-17}\right)\left(2.511 \times 10^{6}\right)} = \dfrac{1.295643}{3.610 \times 2.511} \times 10^{9+17-6} \approx 0.1429 \times 10^{19} = 1.429 \times 10^{19}$

92. $\dfrac{(73.1)\left(1.6341 \times 10^{28}\right)}{0.0000000019} = \dfrac{(7.31 \times 10)\left(1.6341 \times 10^{28}\right)}{1.9 \times 10^{-9}} = \dfrac{7.31 \times 1.6341}{1.9} \times 10^{1+28-(-9)} \approx 6.3 \times 10^{38}$

93. $\dfrac{(0.0000162)(0.01582)}{(594621000)(0.0058)} = \dfrac{\left(1.62 \times 10^{-5}\right)\left(1.582 \times 10^{-2}\right)}{\left(5.94621 \times 10^{8}\right)\left(5.8 \times 10^{-3}\right)} = \dfrac{1.62 \times 1.582}{5.94621 \times 5.8} \times 10^{-5-2-8+3} = 0.074 \times 10^{-12}$
$= 7.4 \times 10^{-14}$

94. $\dfrac{\left(3.542 \times 10^{-6}\right)^{9}}{\left(5.05 \times 10^{4}\right)^{12}} = \dfrac{(3.542)^{9} \times 10^{-54}}{(5.05)^{12} \times 10^{48}} = \dfrac{87747.96}{275103767.10} \times 10^{-54-48} \approx 3.19 \times 10^{-4} \times 10^{-102} \approx 3.19 \times 10^{-106}$

95. **(a)** b^5 is negative since a negative number raised to an odd power is negative.

(b) b^{10} is positive since a negative number raised to an even power is positive.

(c) ab^2c^3 we have (positive) (negative)^2 (negative)^3 = (positive) (positive) (negative) which is negative.

(d) Since $b - a$ is negative, $(b-a)^3 = (\text{negative})^3$ which is negative.

(e) Since $b - a$ is negative, $(b-a)^4 = (\text{negative})^4$ which is positive.

(f) $\dfrac{a^3c^3}{b^6c^6} = \dfrac{(\text{positive})^3\,(\text{negative})^3}{(\text{negative})^6\,(\text{negative})^6} = \dfrac{(\text{positive})\,(\text{negative})}{(\text{positive})\,(\text{positive})} = \dfrac{\text{negative}}{\text{positive}}$ which is negative.

96. **(a)** Since $\frac{1}{2} > \frac{1}{3}$, $2^{1/2} > 2^{1/3}$.

(b) $\left(\frac{1}{2}\right)^{1/2} = 2^{-1/2}$ and $\left(\frac{1}{2}\right)^{1/3} = 2^{-1/3}$. Since $-\frac{1}{2} < -\frac{1}{3}$, we have $\left(\frac{1}{2}\right)^{1/2} < \left(\frac{1}{2}\right)^{1/3}$.

(c) We find a common root: $7^{1/4} = 7^{3/12} = \left(7^3\right)^{1/12} = 343^{1/12}$; $4^{1/3} = 4^{4/12} = \left(4^4\right)^{1/12} = 256^{1/12}$. So $7^{1/4} > 4^{1/3}$.

(d) We find a common root: $\sqrt[3]{5} = 5^{1/3} = 5^{2/6} = \left(5^2\right)^{1/6} = 25^{1/6}$; $\sqrt{3} = 3^{1/2} = 3^{3/6} = \left(3^3\right)^{1/6} = 27^{1/6}$. So $\sqrt[3]{5} < \sqrt{3}$.

97. Since one light year is 5.9×10^{12} miles, Centauri is about $4.3 \times 5.9 \times 10^{12} \approx 2.54 \times 10^{13}$ miles away or 25,400,000,000,000 miles away.

98. $9.3 \times 10^7 \text{ mi} = 186,000 \dfrac{\text{mi}}{\text{s}} \times t \text{ s} \Leftrightarrow t = \dfrac{9.3 \times 10^7}{186,000} \text{ s} = 500 \text{ s} = 8\frac{1}{3} \text{ min}.$

99. Volume = (average depth) (area) = $\left(3.7 \times 10^3 \text{ m}\right)\left(3.6 \times 10^{14} \text{ m}^2\right)\left(\dfrac{10^3 \text{ liters}}{\text{m}^3}\right) \approx 1.33 \times 10^{21}$ liters

100. Each person's share is equal to $\dfrac{\text{national debt}}{\text{population}} = \dfrac{1.674 \times 10^{13}}{3.164 \times 10^{8}} \approx 5.2908 \times 10^4 \approx \$52{,}900$.

101. The number of molecules is equal to

$$(\text{volume}) \cdot \left(\frac{\text{liters}}{\text{m}^3}\right) \cdot \left(\frac{\text{molecules}}{22.4 \text{ liters}}\right) = (5 \cdot 10 \cdot 3) \cdot \left(10^3\right) \cdot \left(\frac{6.02 \times 10^{23}}{22.4}\right) \approx 4.03 \times 10^{27}$$

102. First convert 1135 feet to miles. This gives $1135 \text{ ft} = 1135 \cdot \dfrac{1 \text{ mile}}{5280 \text{ feet}} = 0.215$ mi. Thus the distance you can see is given by $D = \sqrt{2rh + h^2} = \sqrt{2\,(3960)\,(0.215) + (0.215)^2} \approx \sqrt{1702.8} \approx 41.3$ miles.

103. **(a)** Using $f = 0.4$ and substituting $d = 65$, we obtain $s = \sqrt{30fd} = \sqrt{30 \times 0.4 \times 65} \approx 28$ mi/h.

(b) Using $f = 0.5$ and substituting $s = 50$, we find d. This gives $s = \sqrt{30fd} \Leftrightarrow 50 = \sqrt{30 \cdot (0.5)\,d} \Leftrightarrow 50 = \sqrt{15d} \Leftrightarrow 2500 = 15d \Leftrightarrow d = \frac{500}{3} \approx 167$ feet.

104. Since 1 day = 86,400 s, 365.25 days = 31,557,600 s. Substituting, we obtain $d = \left(\dfrac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{4\pi^2}\right)^{1/3} \cdot \left(3.15576 \times 10^7\right)^{2/3} \approx 1.5 \times 10^{11}$ m $= 1.5 \times 10^8$ km.

105. Since $10^6 = 10^3 \cdot 10^3$ it would take 1000 days ≈ 2.74 years to spend the million dollars.
Since $10^9 = 10^3 \cdot 10^6$ it would take $10^6 = 1{,}000{,}000$ days ≈ 2739.72 years to spend the billion dollars.

106. (a) $\dfrac{18^5}{9^5} = \left(\dfrac{18}{9}\right)^5 = 2^5 = 32$

(b) $20^6 \cdot (0.5)^6 = (20 \cdot 0.5)^6 = 10^6 = 1{,}000{,}000$

107. (a)

n	1	2	5	10	100
$2^{1/n}$	$2^{1/1} = 2$	$2^{1/2} = 1.414$	$2^{1/5} = 1.149$	$2^{1/10} = 1.072$	$2^{1/100} = 1.007$

So when n gets large, $2^{1/n}$ decreases to 1.

(b)

n	1	2	5	10	100
$\left(\frac{1}{2}\right)^{1/n}$	$\left(\frac{1}{2}\right)^{1/1} = 0.5$	$\left(\frac{1}{2}\right)^{1/2} = 0.707$	$\left(\frac{1}{2}\right)^{1/5} = 0.871$	$\left(\frac{1}{2}\right)^{1/10} = 0.933$	$\left(\frac{1}{2}\right)^{1/100} = 0.993$

So when n gets large, $\left(\frac{1}{2}\right)^{1/n}$ increases to 1.

108. (a) $\dfrac{a^m}{a^n} = \dfrac{\overbrace{a \cdot a \cdot \cdots \cdot a}^{m \text{ factors}}}{\underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}}$. Because $m > n$, we can cancel n factors of a from numerator and denominator and are left with $m - n$ factors of a in the numerator. Thus, $\dfrac{a^m}{a^n} = a^{m-n}$.

(b) $\left(\dfrac{a}{b}\right)^n = \overbrace{\dfrac{a}{b} \cdot \dfrac{a}{b} \cdot \cdots \cdot \dfrac{a}{b}}^{n \text{ factors}} = \dfrac{\overbrace{a \cdot a \cdot \cdots \cdot a}^{n \text{ factors}}}{\underbrace{b \cdot b \cdot \cdots \cdot b}_{n \text{ factors}}} = \dfrac{a^n}{b^n}$

109. (a) $\left(\dfrac{a}{b}\right)^{-n} = \dfrac{1}{\left(\dfrac{a}{b}\right)^n} = \dfrac{1}{\dfrac{a^n}{b^n}} = \dfrac{b^n}{a^n}$

(b) $\dfrac{a^{-n}}{b^{-m}} = \dfrac{\dfrac{1}{a^n}}{\dfrac{1}{b^m}} = \dfrac{1}{a^n} \cdot b^m = \dfrac{b^m}{a^n}$

1.3 ALGEBRAIC EXPRESSIONS

1. (a) The polynomial $2x^5 + 6x^4 + 4x^3$ has three terms: $2x^5$, $6x^4$, and $4x^3$.

(b) The factor $2x^3$ is common to each term, so $2x^5 + 6x^4 + 4x^3 = 2x^3\left(x^2 + 3x + 2\right)$. [In fact, the polynomial can be factored further as $2x^3(x + 2)(x + 1)$.]

2. To factor the trinomial $x^2 + 7x + 10$ we look for two integers whose product is 10 and whose sum is 7. These integers are 5 and 2, so the trinomial factors as $(x + 5)(x + 2)$.

3. The greatest common factor in the expression $3x^3 + x^2$ is x^2, and the expression factors as $x^2(3x + 1)$.

4. The Special Product Formula for the "square of a sum" is $(A+B)^2 = A^2 + 2AB + B^2$. So $(2x+3)^2 = (2x)^2 + 2(2x)(3) + 3^2 = 4x^2 + 12x + 9$.

5. The Special Product Formula for the "product of the sum and difference of terms" is $(A+B)(A-B) = A^2 - B^2$. So $(5+x)(5-x) = 5^2 - x^2 = 25 - x^2$.

6. The Special Factoring Formula for the "difference of squares" is $A^2 - B^2 = (A-B)(A+B)$. So $4x^2 - 25 = (2x-5)(2x+5)$.

7. The Special Factoring Formula for a "perfect square" is $A^2 + 2AB + B^2 = (A+B)^2$. So $x^2 + 10x + 25 = (x+5)^2$.

8. (a) No; $(x+5)^2 = x^2 + 2(5x) + 25 \neq x^2 + 25$.

(b) Yes; $(x+a)^2 = x^2 + 2xa + a^2$.

(c) Yes; by a Special Product Formula, $(x+5)(x-5) = x^2 - 25$.

(d) Yes, $(x+a)(x-a) = x^2 - a^2$, by a Special Product Formula.

9. Type: binomial. Terms: $5x^3$ and 6. Degree: 3.

10. Type: trinomial. Terms: $-2x^2$, $5x$, and -3. Degree: 2.

11. Type: monomial. Term: -8. Degree: 0.

12. Type: monomial. Terms: $\frac{1}{2}x^7$. Degree: 7.

13. Type: four-term polynomial. Terms: x, $-x^2$, x^3, and $-x^4$. Degree: 4.

14. Type: binomial. Terms: $\sqrt{2}x$ and $-\sqrt{3}$. Degree: 1.

15. $(12x-7) - (5x-12) = 12x - 7 - 5x + 12 = 7x + 5$

16. $(5-3x) + (2x-8) = -x - 3$

17. $\left(-2x^2 - 3x + 1\right) + \left(3x^2 + 5x - 4\right) = -2x^2 - 3x + 1 + 3x^2 + 5x - 4 = x^2 + 2x - 3$

18. $\left(3x^2 + x + 1\right) - \left(2x^2 - 3x - 5\right) = 3x^2 + x + 1 - 2x^2 + 3x + 5 = x^2 + 4x + 6$

19. $\left(5x^3 + 4x^2 - 3x\right) - \left(x^2 + 7x + 2\right) = 5x^3 + 4x^2 - 3x - x^2 - 7x - 2 = 5x^3 + 3x^2 - 10x - 2$

20. $3(x-1) + 4(x+2) = 3x - 3 + 4x + 8 = 7x + 5$

21. $8(2x+5) - 7(x-9) = 16x + 40 - 7x + 63 = 9x + 103$

22. $4\left(x^2 - 3x + 5\right) - 3\left(x^2 - 2x + 1\right) = 4x^2 - 12x + 20 - 3x^2 + 6x - 3 = x^2 - 6x + 17$

23. $2(2-5t) + t^2(t-1) - \left(t^4 - 1\right) = 4 - 10t + t^3 - t^2 - t^4 + 1 = -t^4 + t^3 - t^2 - 10t + 5$

24. $5(3t-4) - \left(t^2 + 2\right) - 2t(t-3) = 15t - 20 - t^2 - 2 - 2t^2 + 6t = -3t^2 + 21t - 22$

25. $(3t-2)(7t-4) = 21t^2 - 12t - 14t + 8 = 21t^2 - 26t + 8$

26. $(4s-1)(2s+5) = 8s^2 + 18s - 5$

27. $(3x+5)(2x-1) = 6x^2 + 10x - 3x - 5 = 6x^2 + 7x - 5$

28. $(7y-3)(2y-1) = 14y^2 - 13y + 3$

29. $(x+3y)(2x-y) = 2x^2 + 5xy - 3y^2$

30. $(4x-5y)(3x-y) = 12x^2 - 19xy + 5y^2$

31. $(5x+1)^2 = 25x^2 + 10x + 1$

32. $(2-7y)^2 = 49y^2 - 28y + 4$

33. $(2u+v)^2 = 4u^2 + 4uv + v^2$

34. $(x-3y)^2 = x^2 - 6xy + 9y^2$

35. $(2x+3y)^2 = 4x^2 + 12xy + 9y^2$

36. $(r-2s)^2 = r^2 - 4rs + 4s^2$

37. $(x+6)(x-6) = x^2 - 36$

38. $(5-y)(5+y) = 25 - y^2$

39. $(3x-4)(3x+4) = (3x)^2 - 4^2 = 9x^2 - 16$

40. $(2y+5)(2y-5) = 4y^2 - 25$

41. $(\sqrt{x}+2)(\sqrt{x}-2) = x - 4$

42. $\left(\sqrt{y}+\sqrt{2}\right)\left(\sqrt{y}-\sqrt{2}\right) = y - 2$

43. $(y+2)^3 = y^3 + 3y^2(2) + 3y\left(2^2\right) + 2^3 = y^3 + 6y^2 + 12y + 8$

44. $(x-3)^3 = x^3 - 9x^2 + 27x - 27$

45. $(1-2r)^3 = 1^3 - 3\left(1^2\right)(2r) + 3(1)(2r)^2 - (2r)^3 = -8r^3 + 12r^2 - 6r + 1$

46. $(3+2y)^3 = 8y^3 + 36y^2 + 54y + 27$

47. $(x+2)\left(x^2+2x+3\right) = x^3 + 2x^2 + 3x + 2x^2 + 4x + 6 = x^3 + 4x^2 + 7x + 6$

48. $(x+1)\left(2x^2-x+1\right) = 2x^3 + x^2 + 1$

49. $(2x-5)\left(x^2-x+1\right) = 2x^3 - 2x^2 + 2x - 5x^2 + 5x - 5 = 2x^3 - 7x^2 + 7x - 5$

50. $(1+2x)\left(x^2-3x+1\right) = 2x^3 - 5x^2 - x + 1$

51. $\sqrt{x}\left(x-\sqrt{x}\right) = x\sqrt{x} - \left(\sqrt{x}\right)^2 = x\sqrt{x} - x$

52. $x^{3/2}\left(\sqrt{x} - 1/\sqrt{x}\right) = x^2 - x$

53. $y^{1/3}\left(y^{2/3}+y^{5/3}\right) = y^{1/3+2/3} + y^{1/3+5/3} = y^2 + y$

54. $x^{1/4}\left(2x^{3/4} - x^{1/4}\right) = 2x - \sqrt{x}$

55. $\left(x^2-a^2\right)\left(x^2+a^2\right) = x^4 - a^4$

56. $\left(x^{1/2}+y^{1/2}\right)\left(x^{1/2}-y^{1/2}\right) = x - y$

57. $\left(\sqrt{a}-b\right)\left(\sqrt{a}+b\right) = a - b^2$

58. $\left(\sqrt{h^2+1}+1\right)\left(\sqrt{h^2+1}-1\right) = h^2$

59. $\left((x-1)+x^2\right)\left((x-1)-x^2\right) = (x-1)^2 - \left(x^2\right)^2 = x^2 - 2x + 1 - x^4 = -x^4 + x^2 - 2x + 1$

60. $\left(x+\left(2+x^2\right)\right)\left(x-\left(2+x^2\right)\right) = -x^4 - 3x^2 - 4$

61. $(2x+y-3)(2x+y+3) = (2x+y)^2 - 3^2 = 4x^2 + 4xy + y^2 - 9$

62. $(x+y+z)(x-y-z) = x^2 - y^2 - z^2 - 2yz$

63. $-2x^3 + x = x\left(1-2x^2\right)$

64. $3x^4 - 6x^3 - x^2 = x^2\left(3x^2-6x-1\right)$

65. $y(y-6) + 9(y-6) = (y-6)(y+9)$

66. $(z+2)^2 - 5(z+2) = (z+2)[(z+2)-5] = (z+2)(z-3)$

67. $2x^2y - 6xy^2 + 3xy = xy(2x-6y+3)$

68. $-7x^4y^2 + 14xy^3 + 21xy^4 = 7xy^2\left(-x^3+2y+3y^2\right)$

69. $x^2 + 8x + 7 = (x+7)(x+1)$

70. $x^2 + 4x - 5 = (x+5)(x-1)$

71. $8x^2 - 14x - 15 = (2x-5)(4x+3)$

72. $6y^2 + 11y - 21 = (y+3)(6y-7)$

73. $3x^2 - 16x + 5 = (3x-1)(x-5)$

74. $5x^2 - 7x - 6 = (5x+3)(x-2)$

75. $(3x+2)^2 + 8(3x+2) + 12 = [(3x+2)+2][(3x+2)+6] = (3x+4)(3x+8)$

76. $2(a+b)^2 + 5(a+b) - 3 = [(a+b)+3][2(a+b)-1] = (a+b+3)(2a+2b-1)$

77. $9a^2 - 16 = (3a)^2 - 4^2 = (3a-4)(3a+4)$

78. $(x+3)^2 - 4 = (x+3)^2 - 2^2 = [(x+3)-2][(x+3)+2] = (x+1)(x+5)$

79. $27x^3 + y^3 = (3x)^3 + y^3 = (3x+y)\left[(3x)^2 + 3xy + y^2\right] = (3x+y)\left(9x^2 - 3xy + y^2\right)$

80. $a^3 - b^6 = a^3 - \left(b^2\right)^3 = \left(a-b^2\right)\left[a^2 + ab^2 + \left(b^2\right)^2\right] = \left(a-b^2\right)\left(a^2+ab^2+b^4\right)$

81. $8s^3 - 125t^3 = (2s)^3 - (5t)^3 = (2s-5t)\left[(2s)^2 + (2s)(5t) + (5t)^2\right] = (2s-5t)\left(4s^2 + 10st + 25t^2\right)$

82. $1 + 1000y^3 = 1 + (10y)^3 = (1+10y)\left[1 - 10y + (10y)^2\right] = (1+10y)\left(1 - 10y + 100y^2\right)$

83. $x^2+12x+36=x^2+2(6x)+6^2=(x+6)^2$

84. $16z^2-24z+9=(4z)^2-2(4z)(3)+3^2=(4z-3)^2$

85. $x^3+4x^2+x+4=x^2(x+4)+1(x+4)=(x+4)\left(x^2+1\right)$

86. $3x^3-x^2+6x-2=x^2(3x-1)+2(3x-1)=(3x-1)\left(x^2+2\right)$

87. $5x^3+x^2+5x+1=x^2(5x+1)+(5x+1)=\left(x^2+1\right)(5x+1)$

88. $18x^3+9x^2+2x+1=9x^2(2x+1)+(2x+1)=\left(9x^2+1\right)(2x+1)$

89. $x^3+x^2+x+1=x^2(x+1)+1(x+1)=(x+1)\left(x^2+1\right)$

90. $x^5+x^4+x+1=x^4(x+1)+1(x+1)=(x+1)\left(x^4+1\right)$

91. $x^{5/2}-x^{1/2}=x^{1/2}\left(x^2-1\right)=\sqrt{x}(x-1)(x+1)$

92. $3x^{-1/2}+4x^{1/2}+x^{3/2}=x^{-1/2}\left(3+4x+x^2\right)=\left(\dfrac{1}{\sqrt{x}}\right)(3+x)(1+x)$

93. Start by factoring out the power of x with the smallest exponent, that is, $x^{-3/2}$. So

$$x^{-3/2}+2x^{-1/2}+x^{1/2}=x^{-3/2}\left(1+2x+x^2\right)=\frac{(1+x)^2}{x^{3/2}}.$$

94. $(x-1)^{7/2}-(x-1)^{3/2}=(x-1)^{3/2}\left[(x-1)^2-1\right]=(x-1)^{3/2}[(x-1)-1][(x-1)+1]$

$$=(x-1)^{3/2}(x-2)(x)$$

95. Start by factoring out the power of $\left(x^2+1\right)$ with the smallest exponent, that is, $\left(x^2+1\right)^{-1/2}$.

Thus, $\left(x^2+1\right)^{1/2}+2\left(x^2+1\right)^{-1/2}=\left(x^2+1\right)^{-1/2}\left[\left(x^2+1\right)+2\right]=\dfrac{x^2+3}{\sqrt{x^2+1}}$.

96. $x^{-1/2}(x+1)^{1/2}+x^{1/2}(x+1)^{-1/2}=x^{-1/2}(x+1)^{-1/2}[(x+1)+x]=\dfrac{2x+1}{\sqrt{x}\sqrt{x+1}}$

97. $12x^3+18x=6x\left(2x^2+3\right)$

98. $30x^3+15x^4=15x^3(2+x)$

99. $x^2-2x-8=(x-4)(x+2)$

100. $x^2-14x+48=(x-8)(x-6)$

101. $2x^2+5x+3=(2x+3)(x+1)$

102. $2x^2+7x-4=(2x-1)(x+4)$

103. $9x^2-36x-45=9\left(x^2-4x-5\right)=9(x-5)(x+1)$

104. $8x^2+10x+3=(4x+3)(2x+1)$

105. $49-4y^2=(7-2y)(7+2y)$

106. $4t^2-9s^2=(2t-3s)(2t+3s)$

107. $t^2-6t+9=(t-3)^2$

108. $x^2+10x+25=(x+5)^2$

109. $4x^2+4xy+y^2=(2x+y)^2$

110. $r^2-6rs+9s^2=(r-3s)^2$

111. $(a+b)^2-(a-b)^2=[(a+b)-(a-b)][(a+b)+(a-b)]=(2b)(2a)=4ab$

112. $\left(1+\dfrac{1}{x}\right)^2-\left(1-\dfrac{1}{x}\right)^2=\left[\left(1+\dfrac{1}{x}\right)-\left(1-\dfrac{1}{x}\right)\right]\left[\left(1+\dfrac{1}{x}\right)+\left(1-\dfrac{1}{x}\right)\right]$

$$=\left(1+\frac{1}{x}-1+\frac{1}{x}\right)\left(1+\frac{1}{x}+1-\frac{1}{x}\right)=\left(\frac{2}{x}\right)(2)=\frac{4}{x}$$

113. $x^2\left(x^2-1\right)-9\left(x^2-1\right)=\left(x^2-1\right)\left(x^2-9\right)=(x-1)(x+1)(x-3)(x+3)$

114. $\left(a^2-1\right)b^2-4\left(a^2-1\right)=\left(a^2-1\right)\left(b^2-4\right)=(a-1)(a+1)(b-2)(b+2)$

115. $8x^3 - 125 = (2x)^3 - 5^3 = (2x-5)\left[(2x)^2 + (2x)(5) + 5^2\right] = (2x-5)\left(4x^2 + 10x + 25\right)$

116. $x^6 + 64 = x^6 + 2^6 = \left(x^2\right)^3 + (4)^3 = \left(x^2+4\right)\left[\left(x^2\right)^2 - 4\left(x^2\right) + (4)^2\right] = \left(x^2+4\right)\left(x^4 - 4x^2 + 16\right)$

117. $x^3 + 2x^2 + x = x\left(x^2 + 2x + 1\right) = x(x+1)^2$

118. $3x^3 - 27x = 3x\left(x^2 - 9\right) = 3x(x-3)(x+3)$

119. $x^4y^3 - x^2y^5 = x^2y^3\left(x^2 - y^2\right) = x^2y^3(x+y)(x-y)$

120. $18y^3x^2 - 2xy^4 = 2xy^3(9x - y)$

121. $3x^3 - x^2 - 12x + 4 = 3x\left(x^2-4\right) - \left(x^2-4\right) = (3x-1)\left(x^2-4\right) = (3x-1)(x+2)(x-2)$

122. $9x^3 + 18x^2 - x - 2 = 9x^2(x+2) - (x+2) = \left(9x^2-1\right)(x+2) = (3x+1)(3x-1)(x+2)$

123. $(x-1)(x+2)^2 - (x-1)^2(x+2) = (x-1)(x+2)[(x+2)-(x-1)] = 3(x-1)(x+2)$

124. $y^4(y+2)^3 + y^5(y+2)^4 = y^4(y+2)^3[(1) + y(y+2)] = y^4(y+2)^3\left(y^2+2y+1\right) = y^4(y+2)^3(y+1)^2$

125. Start by factoring $y^2 - 7y + 10$, and then substitute $a^2 + 1$ for y. This gives
$\left(a^2+1\right)^2 - 7\left(a^2+1\right) + 10 = \left[\left(a^2+1\right) - 2\right]\left[\left(a^2+1\right) - 5\right] = \left(a^2-1\right)\left(a^2-4\right) = (a-1)(a+1)(a-2)(a+2)$

126. $\left(a^2+2a\right)^2 - 2\left(a^2+2a\right) - 3 = \left[\left(a^2+2a\right) - 3\right]\left[\left(a^2+2a\right) + 1\right] = \left(a^2+2a-3\right)\left(a^2+2a+1\right)$
$= (a-1)(a+3)(a+1)^2$

127. $5\left(x^2+4\right)^4(2x)(x-2)^4 + \left(x^2+4\right)^5(4)(x-2)^3 = 2\left(x^2+4\right)^4(x-2)^3\left[(5)(x)(x-2) + \left(x^2+4\right)(2)\right]$
$= 2\left(x^2+4\right)^4(x-2)^3\left(5x^2 - 10x + 2x^2 + 8\right) = 2\left(x^2+4\right)^4(x-2)^3\left(7x^2 - 10x + 8\right)$

128. $3(2x-1)^2(2)(x+3)^{1/2} + (2x-1)^3\left(\frac{1}{2}\right)(x+3)^{-1/2} = (2x-1)^2(x+3)^{-1/2}\left[6(x+3) + (2x-1)\left(\frac{1}{2}\right)\right]$
$= (2x-1)^2(x+3)^{-1/2}\left(6x + 18 + x - \frac{1}{2}\right) = (2x-1)^2(x+3)^{-1/2}\left(7x + \frac{35}{2}\right)$

129. $\left(x^2+3\right)^{-1/3} - \frac{2}{3}x^2\left(x^2+3\right)^{-4/3} = \left(x^2+3\right)^{-4/3}\left[\left(x^2+3\right) - \frac{2}{3}x^2\right] = \left(x^2+3\right)^{-4/3}\left(\frac{1}{3}x^2 + 3\right) = \dfrac{\frac{1}{3}x^2 + 3}{\left(x^2+3\right)^{4/3}}$

130. $\frac{1}{2}x^{-1/2}(3x+4)^{1/2} - \frac{3}{2}x^{1/2}(3x+4)^{-1/2} = \frac{1}{2}x^{-1/2}(3x+4)^{-1/2}[(3x+4) - 3x] = \frac{1}{2}x^{-1/2}(3x+4)^{-1/2}(4)$
$= 2x^{-1/2}(3x+4)^{-1/2}$

131. **(a)** $\frac{1}{2}\left[(a+b)^2 - \left(a^2+b^2\right)\right] = \frac{1}{2}\left[a^2 + 2ab + b^2 - a^2 - b^2\right] = \frac{1}{2}(2ab) = ab.$

(b) $\left(a^2+b^2\right)^2 - \left(a^2-b^2\right)^2 = \left[\left(a^2+b^2\right) - \left(a^2-b^2\right)\right]\left[\left(a^2+b^2\right) + \left(a^2-b^2\right)\right]$
$= \left(a^2 + b^2 - a^2 + b^2\right)\left(a^2 + b^2 + a^2 - b^2\right) = \left(2b^2\right)\left(2a^2\right) = 4a^2b^2$

132. LHS $= \left(a^2+b^2\right)\left(c^2+d^2\right) = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2.$
RHS $= (ac+bd)^2 + (ad-bc)^2 = a^2c^2 + 2abcd + b^2d^2 + a^2d^2 - 2abcd + b^2c^2 = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2.$
So LHS = RHS, that is, $\left(a^2+b^2\right)\left(c^2+d^2\right) = (ac+bd)^2 + (ad-bc)^2.$

133. $4a^2c^2 - \left(c^2 - b^2 + a^2\right)^2 = (2ac)^2 - \left(c^2 - b^2 + a^2\right)$

$= \left[(2ac) - \left(c^2 - b^2 + a^2\right)\right]\left[(2ac) + \left(c^2 - b^2 + a^2\right)\right]$ (difference of squares)

$= \left(2ac - c^2 + b^2 - a^2\right)\left(2ac + c^2 - b^2 + a^2\right)$

$= \left[b^2 - \left(c^2 - 2ac + a^2\right)\right]\left[\left(c^2 + 2ac + a^2\right) - b^2\right]$ (regrouping)

$= \left[b^2 - (c - a)^2\right]\left[(c + a)^2 - b^2\right]$ (perfect squares)

$= [b - (c - a)][b + (c - a)][(c + a) - b][(c + a) + b]$ (each factor is a difference of squares)

$= (b - c + a)(b + c - a)(c + a - b)(c + a + b)$

$= (a + b - c)(-a + b + c)(a - b + c)(a + b + c)$

134. **(a)** $x^4 + x^2 - 2 = \left(x^2 - 1\right)\left(x^2 + 2\right) = (x - 1)(x + 1)\left(x^2 + 2\right)$

(b) $x^4 + 2x^2 + 9 = \left(x^4 + 6x^2 + 9\right) - 4x^2 = \left(x^2 + 3\right)^2 - (2x)^2 = \left[\left(x^2 + 3\right) - 2x\right]\left[\left(x^2 + 3\right) + 2x\right]$

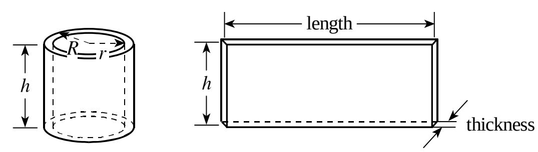
$= \left(x^2 - 2x + 3\right)\left(x^2 + 2x + 3\right)$

(c) $x^4 + 4x^2 + 16 = \left(x^4 + 8x^2 + 16\right) - 4x^2 = \left(x^2 + 4\right)^2 - (2x)^2$

$= \left[\left(x^2 + 4\right) - 2x\right]\left[\left(x^2 + 4\right) + 2x\right] = \left(x^2 - 2x + 4\right)\left(x^2 + 2x + 4\right)$

(d) $x^4 + 2x^2 + 1 = \left(x^2 + 1\right)^2$

135. The volume of the shell is the difference between the volumes of the outside cylinder (with radius R) and the inside cylinder (with radius r). Thus $V = \pi R^2 h - \pi r^2 h = \pi\left(R^2 - r^2\right) h = \pi (R - r)(R + r) h = 2\pi \cdot \dfrac{R + r}{2} \cdot h \cdot (R - r)$. The average radius is $\dfrac{R + r}{2}$ and $2\pi \cdot \dfrac{R + r}{2}$ is the average circumference (length of the rectangular box), h is the height, and $R - r$ is the thickness of the rectangular box. Thus $V = \pi R^2 h - \pi r^2 h = 2\pi \cdot \dfrac{R + r}{2} \cdot h \cdot (R - r) = 2\pi \cdot$ (average radius) $\cdot$ (height) $\cdot$ (thickness)



136. **(a)** Moved portion = field − habitat

(b) Using the difference of squares, we get $b^2 - (b - 2x)^2 = [b - (b - 2x)][b + (b - x)] = 2x(2b - 2x) = 4x(b - x)$.

137. **(a)** The degree of the product is the sum of the degrees.

(b) The degree of a sum is at most the largest of the degrees — it could be smaller than either. For example, the degree of $\left(x^3\right) + \left(-x^3 + x\right) = x$ is 1.

138. **(a)** $528^2 - 527^2 = (528 - 527)(528 + 527) = 1(1055) = 1055$

(b) $122^2 - 120^2 = (122 - 120)(122 + 120) = 2(242) = 484$

(c) $1020^2 - 1010^2 = (1020 - 1010)(1020 + 1010) = 10(2030) = 20{,}300$

139. **(a)** $501 \cdot 499 = (500 + 1)(500 - 1) = 500^2 - 1 = 250{,}000 - 1 = 249{,}999$

(b) $79 \cdot 61 = (70 + 9)(70 - 9) = 70^2 - 9^2 = 4900 - 81 = 4819$

(c) $2007 \cdot 1993 = (2000 + 7)(2000 - 7) = 2000^2 - 7^2 = 4{,}000{,}000 - 49 = 3{,}999{,}951$

140. **(a)** $A^4 - B^4 = \left(A^2 - B^2\right)\left(A^2 + B^2\right) = (A - B)(A + B)\left(A^2 + B^2\right)$

$A^6 - B^6 = \left(A^3 - B^3\right)\left(A^3 + B^3\right)$ (difference of squares)

$= (A - B)\left(A^2 + AB + B^2\right)(A + B)\left(A^2 - AB + B^2\right)$ (difference and sum of cubes)

(b) $12^4 - 7^4 = 20{,}736 - 2{,}401 = 18{,}335$; $12^6 - 7^6 = 2{,}985{,}984 - 117{,}649 = 2{,}868{,}335$

(c) $18{,}335 = 12^4 - 7^4 = (12 - 7)(12 + 7)\left(12^2 + 7^2\right) = 5(19)(144 + 49) = 5(19)(193)$

$2{,}868{,}335 = 12^6 - 7^6 = (12 - 7)(12 + 7)\left[12^2 + 12(7) + 7^2\right]\left[12^2 - 12(7) + 7^2\right]$

$= 5(19)(144 + 84 + 49)(144 - 84 + 49) = 5(19)(277)(109)$

141. **(a)** $(A - 1)(A + 1) = A^2 + A - A - 1 = A^2 - 1$

$(A - 1)\left(A^2 + A + 1\right) = A^3 + A^2 + A - A^2 - A - 1 = A^3 - 1$

$(A - 1)\left(A^3 + A^2 + A + 1\right) = A^4 + A^3 + A^2 + A - A^3 - A^2 - A - 1$

(b) We conjecture that $A^5 - 1 = (A - 1)\left(A^4 + A^3 + A^2 + A + 1\right)$. Expanding the right-hand side, we have $(A - 1)\left(A^4 + A^3 + A^2 + A + 1\right) = A^5 + A^4 + A^3 + A^2 + A - A^4 - A^3 - A^2 - A - 1 = A^5 - 1$, verifying our conjecture. Generally, $A^n - 1 = (A - 1)\left(A^{n-1} + A^{n-2} + \cdots + A + 1\right)$ for any positive integer n.

142. **(a)**

$$\begin{array}{rrrr} & & A & +\ 1 \\ \times & & A & -\ 1 \\ \hline & & -A & -\ 1 \\ A^2 & + & A & \\ \hline A^2 & & & -\ 1 \end{array} \qquad \begin{array}{rrrrr} & A^2 & + A & + 1 \\ \times & & A & - 1 \\ \hline & -A^2 & - A & - 1 \\ A^3 + & A^2 & + A & \\ \hline A^3 & & & - 1 \end{array} \qquad \begin{array}{rrrrr} & A^3 + & A^2 & + A & + 1 \\ \times & & & A & - 1 \\ \hline & -A^3 - & A^2 & - A & - 1 \\ A^4 + & A^3 + & A^2 & + A & \\ \hline A^4 & & & & - 1 \end{array}$$

(b) Based on the pattern in part (a), we suspect that $A^5 - 1 = (A - 1)\left(A^4 + A^3 + A^2 + A + 1\right)$. Check:

$$\begin{array}{rrrrrr} & A^4 + & A^3 + & A^2 & + A & + 1 \\ \times & & & & A & - 1 \\ \hline & -A^4 - & A^3 - & A^2 & - A & - 1 \\ A^5 + & A^4 + & A^3 + & A^2 & + A & \\ \hline A^5 & & & & & - 1 \end{array}$$

The general pattern is $A^n - 1 = (A - 1)\left(A^{n-1} + A^{n-2} + \cdots + A^2 + A + 1\right)$, where n is a positive integer.

1.4 RATIONAL EXPRESSIONS

1. **(a)** $\dfrac{3x}{x^2 - 1}$ is a rational expression.

(b) $\dfrac{\sqrt{x + 1}}{2x + 3}$ is not a rational expression. A rational expression must be a polynomial divided by a polynomial, and the numerator of the expression is $\sqrt{x + 1}$, which is not a polynomial.

(c) $\dfrac{x(x^2 - 1)}{x + 3} = \dfrac{x^3 - x}{x + 3}$ is a rational expression.

2. To simplify a rational expression we cancel factors that are common to the *numerator* and *denominator*. So, the expression $\dfrac{(x+1)(x+2)}{(x+3)(x+2)}$ simplifies to $\dfrac{x+1}{x+3}$.

3. To multiply two rational expressions we multiply their *numerators* together and multiply their *denominators* together. So $\dfrac{2}{x+1} \cdot \dfrac{x}{x+3}$ is the same as $\dfrac{2 \cdot x}{(x+1) \cdot (x+3)} = \dfrac{2x}{x^2+4x+3}$.

4. **(a)** $\dfrac{1}{x} - \dfrac{2}{(x+1)} - \dfrac{x}{(x+1)^2}$ has three terms.

(b) The least common denominator of all the terms is $x\,(x+1)^2$.

(c) $$\frac{1}{x} - \frac{2}{(x+1)} - \frac{x}{(x+1)^2} = \frac{(x+1)^2}{x\,(x+1)^2} - \frac{2x\,(x+1)}{(x+1)} - \frac{x\,(x)}{(x+1)^2} = \frac{(x+1)^2 - 2x\,(x+1) - x^2}{x\,(x+1)^2}$$
$$= \frac{x^2+2x+1-2x^2-2x-x^2}{x\,(x+1)^2} = \frac{-2x^2+1}{x\,(x+1)^2}$$

5. **(a)** Yes. Cancelling $x+1$, we have $\dfrac{x\,(x+1)}{(x+1)^2} = \dfrac{x}{x+1}$.

(b) No; $(x+5)^2 = x^2+10x+25 \neq x^2+25$, so $x+5 = \sqrt{x^2+10x+25} \neq \sqrt{x^2+25}$.

6. **(a)** Yes, $\dfrac{3+a}{3} = \dfrac{3}{3} + \dfrac{a}{3} = 1 + \dfrac{a}{3}$.

(b) No. We cannot "separate" the denominator in this way; only the numerator, as in part (a). (See also Exercise 101.)

7. The domain of $4x^2-10x+3$ is all real numbers.

8. The domain of $-x^4+x^3+9x$ is all real numbers.

9. Since $x-3 \neq 0$ we have $x \neq 3$. Domain: $\{x \mid x \neq 3\}$

10. Since $3t+6 \neq 0$ we have $t \neq -2$. Domain: $\{t \mid t \neq -2\}$

11. Since $x+3 \geq 0$, $x \geq -3$. Domain; $\{x \mid x \geq -3\}$

12. Since $x-1 > 0$, $x > 1$. Domain; $\{x \mid x > 1\}$

13. $x^2-x-2 = (x+1)(x-2) \neq 0 \Leftrightarrow x \neq -1$ or 2, so the domain is $\{x \mid x \neq -1, 2\}$.

14. $2x \geq 0$ and $x+1 \neq 0 \Leftrightarrow x \geq 0$ and $x \neq -1$, so the domain is $\{x \mid x \geq 0\}$.

15. $\dfrac{5\,(x-3)\,(2x+1)}{10\,(x-3)^2} = \dfrac{5\,(x-3)\,(2x+1)}{5\,(x-3)\cdot 2\,(x-3)} = \dfrac{2x+1}{2\,(x-3)}$

16. $\dfrac{4\left(x^2-1\right)}{12\,(x+2)\,(x-1)} = \dfrac{4\,(x+1)\,(x-1)}{12\,(x+2)\,(x-1)} = \dfrac{x+1}{3\,(x+2)}$

17. $\dfrac{x-2}{x^2-4} = \dfrac{x-2}{(x-2)\,(x+2)} = \dfrac{1}{x+2}$

18. $\dfrac{x^2-x-2}{x^2-1} = \dfrac{(x-2)\,(x+1)}{(x-1)\,(x+1)} = \dfrac{x-2}{x-1}$

19. $\dfrac{x^2+5x+6}{x^2+8x+15} = \dfrac{(x+2)\,(x+3)}{(x+5)\,(x+3)} = \dfrac{x+2}{x+5}$

20. $\dfrac{x^2-x-12}{x^2+5x+6} = \dfrac{(x-4)\,(x+3)}{(x+2)\,(x+3)} = \dfrac{x-4}{x+2}$

21. $\dfrac{y^2+y}{y^2-1} = \dfrac{y\,(y+1)}{(y-1)\,(y+1)} = \dfrac{y}{y-1}$

22. $\dfrac{y^2-3y-18}{2y^2+7y+3} = \dfrac{(y-6)\,(y+3)}{(2y+1)\,(y+3)} = \dfrac{y-6}{2y+1}$

23. $\dfrac{2x^3-x^2-6x}{2x^2-7x+6} = \dfrac{x\left(2x^2-x-6\right)}{(2x-3)\,(x-2)} = \dfrac{x\,(2x+3)\,(x-2)}{(2x-3)\,(x-2)} = \dfrac{x\,(2x+3)}{2x-3}$

24. $\dfrac{1-x^2}{x^3-1} = \dfrac{(1-x)\,(1+x)}{(x-1)\left(x^2+x+1\right)} = \dfrac{-(x-1)\,(1+x)}{(x-1)\left(x^2+x+1\right)} = \dfrac{-(x+1)}{x^2+x+1}$

25. $\dfrac{4x}{x^2-4} \cdot \dfrac{x+2}{16x} = \dfrac{4x}{(x-2)\,(x+2)} \cdot \dfrac{x+2}{16x} = \dfrac{1}{4\,(x-2)}$

26. $\dfrac{x^2-25}{x^2-16} \cdot \dfrac{x+4}{x+5} = \dfrac{(x-5)\,(x+5)}{(x-4)\,(x+4)} \cdot \dfrac{x+4}{x+5} = \dfrac{x-5}{x-4}$

27. $\dfrac{x^2+2x-15}{x^2-25} \cdot \dfrac{x-5}{x+2} = \dfrac{(x+5)\,(x-3)\,(x-5)}{(x+5)\,(x-5)\,(x+2)} = \dfrac{x-3}{x+2}$

28. $\dfrac{x^2+2x-3}{x^2-2x-3}\cdot\dfrac{3-x}{3+x}=\dfrac{(x+3)(x-1)}{(x-3)(x+1)}\cdot\dfrac{-(x-3)}{x+3}=\dfrac{-(x-1)}{x+1}=\dfrac{-x+1}{x+1}=\dfrac{1-x}{1+x}$

29. $\dfrac{t-3}{t^2+9}\cdot\dfrac{t+3}{t^2-9}=\dfrac{(t-3)(t+3)}{\left(t^2+9\right)(t-3)(t+3)}=\dfrac{1}{t^2+9}$

30. $\dfrac{x^2-x-6}{x^2+2x}\cdot\dfrac{x^3+x^2}{x^2-2x-3}=\dfrac{(x-3)(x+2)}{x(x+2)}\cdot\dfrac{x^2(x+1)}{(x-3)(x+1)}=x$

31. $\dfrac{x^2+7x+12}{x^2+3x+2}\cdot\dfrac{x^2+5x+6}{x^2+6x+9}=\dfrac{(x+3)(x+4)}{(x+1)(x+2)}\cdot\dfrac{(x+2)(x+3)}{(x+3)(x+3)}=\dfrac{x+4}{x+1}$

32. $\dfrac{x^2+2xy+y^2}{x^2-y^2}\cdot\dfrac{2x^2-xy-y^2}{x^2-xy-2y^2}=\dfrac{(x+y)(x+y)}{(x-y)(x+y)}\cdot\dfrac{(x-y)(2x+y)}{(x-2y)(x+y)}=\dfrac{2x+y}{x-2y}$

33. $\dfrac{x+3}{4x^2-9}\div\dfrac{x^2+7x+12}{2x^2+7x-15}=\dfrac{x+3}{4x^2-9}\cdot\dfrac{2x^2+7x-15}{x^2+7x+12}=\dfrac{x+3}{(2x-3)(2x+3)}\cdot\dfrac{(x+5)(2x-3)}{(x+3)(x+4)}=\dfrac{x+5}{(2x+3)(x+4)}$

34. $\dfrac{2x+1}{2x^2+x-15}\div\dfrac{6x^2-x-2}{x+3}=\dfrac{2x+1}{(x+3)(2x-5)}\cdot\dfrac{x+3}{(2x+1)(3x-2)}=\dfrac{1}{(2x-5)(3x-2)}$

35. $\dfrac{\dfrac{x^3}{x+1}}{\dfrac{x}{x^2+2x+1}}=\dfrac{x^3}{x+1}\cdot\dfrac{x^2+2x+1}{x}=\dfrac{x^3(x+1)(x+1)}{(x+1)x}=x^2(x+1)$

36. $\dfrac{\dfrac{2x^2-3x-2}{x^2-1}}{\dfrac{2x^2+5x+2}{x^2+x-2}}=\dfrac{2x^2-3x-2}{x^2-1}\cdot\dfrac{x^2+x-2}{2x^2+5x+2}=\dfrac{(x-2)(2x+1)}{(x-1)(x+1)}\cdot\dfrac{(x-1)(x+2)}{(x+2)(2x+1)}=\dfrac{x-2}{x+1}$

37. $\dfrac{x/y}{z}=\dfrac{x}{y}\cdot\dfrac{1}{z}=\dfrac{x}{yz}$

38. $\dfrac{x}{y/z}=x\div\dfrac{y}{z}=\dfrac{x}{1}\cdot\dfrac{z}{y}=\dfrac{xz}{y}$

39. $1+\dfrac{1}{x+3}=\dfrac{x+3}{x+3}+\dfrac{1}{x+3}=\dfrac{x+4}{x+3}$

40. $\dfrac{3x-2}{x+1}-2=\dfrac{3x-2}{x+1}-\dfrac{2(x+1)}{x+1}=\dfrac{3x-2-2x-2}{x+1}=\dfrac{x-4}{x+1}$

41. $\dfrac{1}{x+5}+\dfrac{2}{x-3}=\dfrac{x-3}{(x+5)(x-3)}+\dfrac{2(x+5)}{(x+5)(x-3)}=\dfrac{x-3+2x+10}{(x+5)(x-3)}=\dfrac{3x+7}{(x+5)(x-3)}$

42. $\dfrac{1}{x+1}+\dfrac{1}{x-1}=\dfrac{x-1}{(x+1)(x-1)}+\dfrac{x+1}{(x+1)(x-1)}=\dfrac{x-1+x+1}{(x+1)(x-1)}=\dfrac{2x}{(x+1)(x-1)}$

43. $\dfrac{3}{x+1}-\dfrac{1}{x+2}=\dfrac{3(x+2)}{(x+1)(x+2)}-\dfrac{1(x+1)}{(x+1)(x+2)}=\dfrac{3x+6-x-1}{(x+1)(x+2)}=\dfrac{2x+5}{(x+1)(x+2)}$

44. $\dfrac{x}{x-4}-\dfrac{3}{x+6}=\dfrac{x(x+6)}{(x-4)(x+6)}+\dfrac{-3(x-4)}{(x-4)(x+6)}=\dfrac{x^2+6x-3x+12}{(x-4)(x+6)}=\dfrac{x^2+3x+12}{(x-4)(x+6)}$

45. $\dfrac{5}{2x-3}-\dfrac{3}{(2x-3)^2}=\dfrac{5(2x-3)}{(2x-3)^2}-\dfrac{3}{(2x-3)^2}=\dfrac{10x-15-3}{(2x-3)^2}=\dfrac{10x-18}{(2x-3)^2}=\dfrac{2(5x-9)}{(2x-3)^2}$

46. $\dfrac{x}{(x+1)^2}+\dfrac{2}{x+1}=\dfrac{x}{(x+1)^2}+\dfrac{2(x+1)}{(x+1)(x+1)}=\dfrac{x+2x+2}{(x+1)^2}=\dfrac{3x+2}{(x+1)^2}$

47. $u+1+\dfrac{u}{u+1}=\dfrac{(u+1)(u+1)}{u+1}+\dfrac{u}{u+1}=\dfrac{u^2+2u+1+u}{u+1}=\dfrac{u^2+3u+1}{u+1}$

48. $\dfrac{2}{a^2}-\dfrac{3}{ab}+\dfrac{4}{b^2}=\dfrac{2b^2}{a^2b^2}-\dfrac{3ab}{a^2b^2}+\dfrac{4a^2}{a^2b^2}=\dfrac{2b^2-3ab+4a^2}{a^2b^2}$

49. $\dfrac{1}{x^2}+\dfrac{1}{x^2+x}=\dfrac{1}{x^2}+\dfrac{1}{x(x+1)}=\dfrac{x+1}{x^2(x+1)}+\dfrac{x}{x^2(x+1)}=\dfrac{2x+1}{x^2(x+1)}$

50. $\dfrac{1}{x}+\dfrac{1}{x^2}+\dfrac{1}{x^3}=\dfrac{x^2}{x^3}+\dfrac{x}{x^3}+\dfrac{1}{x^3}=\dfrac{x^2+x+1}{x^3}$

51. $\dfrac{2}{x+3}-\dfrac{1}{x^2+7x+12}=\dfrac{2}{x+3}-\dfrac{1}{(x+3)(x+4)}=\dfrac{2(x+4)}{(x+3)(x+4)}+\dfrac{-1}{(x+3)(x+4)}$
$=\dfrac{2x+8-1}{(x+3)(x+4)}=\dfrac{2x+7}{(x+3)(x+4)}$

52. $\dfrac{x}{x^2-4}+\dfrac{1}{x-2}=\dfrac{x}{(x-2)(x+2)}+\dfrac{1}{x-2}=\dfrac{x}{(x-2)(x+2)}+\dfrac{x+2}{(x-2)(x+2)}$
$=\dfrac{2x+2}{(x-2)(x+2)}=\dfrac{2(x+1)}{(x-2)(x+2)}$

53. $\dfrac{1}{x+3}+\dfrac{1}{x^2-9}=\dfrac{1}{x+3}+\dfrac{1}{(x-3)(x+3)}=\dfrac{x-3}{(x-3)(x+3)}+\dfrac{1}{(x-3)(x+3)}=\dfrac{x-2}{(x-3)(x+3)}$

54. $\dfrac{x}{x^2+x-2}-\dfrac{2}{x^2-5x+4}=\dfrac{x}{(x-1)(x+2)}+\dfrac{-2}{(x-1)(x-4)}$
$=\dfrac{x(x-4)}{(x-1)(x+2)(x-4)}+\dfrac{-2(x+2)}{(x-1)(x+2)(x-4)}=\dfrac{x^2-4x-2x-4}{(x-1)(x+2)(x-4)}=\dfrac{x^2-6x-4}{(x-1)(x+2)(x-4)}$

55. $\dfrac{2}{x}+\dfrac{3}{x-1}-\dfrac{4}{x^2-x}=\dfrac{2}{x}+\dfrac{3}{x-1}-\dfrac{4}{x(x-1)}=\dfrac{2(x-1)}{x(x-1)}+\dfrac{3x}{x(x-1)}+\dfrac{-4}{x(x-1)}=\dfrac{2x-2+3x-4}{x(x-1)}=\dfrac{5x-6}{x(x-1)}$

56. $\dfrac{x}{x^2-x-6}-\dfrac{1}{x+2}-\dfrac{2}{x-3}=\dfrac{x}{(x-3)(x+2)}+\dfrac{-1}{x+2}+\dfrac{-2}{x-3}$
$=\dfrac{x}{(x-3)(x+2)}+\dfrac{-1(x-3)}{(x-3)(x+2)}+\dfrac{-2(x+2)}{(x-3)(x+2)}=\dfrac{x-x+3-2x-4}{(x-3)(x+2)}=\dfrac{-2x-1}{(x-3)(x+2)}$

57. $\dfrac{1}{x^2+3x+2}-\dfrac{1}{x^2-2x-3}=\dfrac{1}{(x+2)(x+1)}-\dfrac{1}{(x-3)(x+1)}$
$=\dfrac{x-3}{(x-3)(x+2)(x+1)}+\dfrac{-(x+2)}{(x-3)(x+2)(x+1)}=\dfrac{x-3-x-2}{(x-3)(x+2)(x+1)}=\dfrac{-5}{(x-3)(x+2)(x+1)}$

58. $\dfrac{1}{x+1}-\dfrac{2}{(x+1)^2}+\dfrac{3}{x^2-1}=\dfrac{1}{x+1}+\dfrac{-2}{(x+1)^2}+\dfrac{3}{(x-1)(x+1)}$
$=\dfrac{(x+1)(x-1)}{(x-1)(x+1)^2}+\dfrac{-2(x-1)}{(x-1)(x+1)^2}+\dfrac{3(x+1)}{(x-1)(x+1)^2}$
$=\dfrac{x^2-1}{(x-1)(x+1)^2}+\dfrac{-2x+2}{(x-1)(x+1)^2}+\dfrac{3x+3}{(x-1)(x+1)^2}=\dfrac{x^2-1-2x+2+3x+3}{(x-1)(x+1)^2}=\dfrac{x^2+x+4}{(x-1)(x+1)^2}$

59. $\dfrac{1+\frac{1}{x}}{\frac{1}{x}-2}=\dfrac{x\left(1+\frac{1}{x}\right)}{x\left(\frac{1}{x}-2\right)}=\dfrac{x+1}{1-2x}$

60. $\dfrac{1-\frac{2}{y}}{\frac{3}{y}-1}=\dfrac{y\left(1-\frac{2}{y}\right)}{y\left(\frac{3}{y}-1\right)}=\dfrac{y-2}{3-y}$

61. $\dfrac{1+\dfrac{1}{x+2}}{1-\dfrac{1}{x+2}}=\dfrac{(x+2)\left(1+\dfrac{1}{x+2}\right)}{(x+2)\left(1-\dfrac{1}{x+2}\right)}=\dfrac{(x+2)+1}{(x+2)-1}=\dfrac{x+3}{x+1}$

62. $\dfrac{1+\dfrac{1}{c-1}}{1-\dfrac{1}{c-1}}=\dfrac{c-1+1}{c-1-1}=\dfrac{c}{c-2}$

63. $\dfrac{\dfrac{1}{x-1}+\dfrac{1}{x+3}}{x+1}=\dfrac{(x-1)(x+3)\left(\dfrac{1}{x-1}+\dfrac{1}{x+3}\right)}{(x-1)(x+3)(x+1)}=\dfrac{(x+3)+(x-1)}{(x-1)(x+1)(x+3)}=\dfrac{2(x+1)}{(x-1)(x+1)(x+3)}$

$=\dfrac{2}{(x-1)(x+3)}$

64. $\dfrac{\dfrac{x-3}{x-4}-\dfrac{x+2}{x+1}}{x+3}=\dfrac{(x-3)(x+1)-(x+2)(x-4)}{(x-4)(x+3)(x+1)}=\dfrac{x^2-2x-3-\left(x^2-2x-8\right)}{(x-4)(x+3)(x+1)}=\dfrac{5}{(x-4)(x+3)(x+1)}$

65. $\dfrac{x-\dfrac{x}{y}}{y-\dfrac{y}{x}}=\dfrac{xy\left(x-\dfrac{x}{y}\right)}{xy\left(y-\dfrac{y}{x}\right)}=\dfrac{x^2y-x^2}{xy^2-y^2}=\dfrac{x^2(y-1)}{y^2(x-1)}$

66. $\dfrac{x+\dfrac{y}{x}}{y+\dfrac{x}{y}}=\dfrac{xy\left(x+\dfrac{y}{x}\right)}{xy\left(y+\dfrac{x}{y}\right)}=\dfrac{x^2y+y^2}{xy^2+x^2}=\dfrac{y\left(y+x^2\right)}{x\left(x+y^2\right)}$

67. $\dfrac{\dfrac{x}{y}-\dfrac{y}{x}}{\dfrac{1}{x^2}-\dfrac{1}{y^2}}=\dfrac{\dfrac{x^2-y^2}{xy}}{\dfrac{y^2-x^2}{x^2y^2}}=\dfrac{x^2-y^2}{xy}\cdot\dfrac{x^2y^2}{y^2-x^2}=\dfrac{xy}{-1}=-xy$. An alternative method is to multiply the numerator and denominator by the common denominator of both the numerator and denominator, in this case x^2y^2:

$\dfrac{\dfrac{x}{y}-\dfrac{y}{x}}{\dfrac{1}{x^2}-\dfrac{1}{y^2}}=\dfrac{\left(\dfrac{x}{y}-\dfrac{y}{x}\right)}{\left(\dfrac{1}{x^2}-\dfrac{1}{y^2}\right)}\cdot\dfrac{x^2y^2}{x^2y^2}=\dfrac{x^3y-xy^3}{y^2-x^2}=\dfrac{xy\left(x^2-y^2\right)}{y^2-x^2}=-xy.$

68. $x-\dfrac{y}{\dfrac{x}{y}+\dfrac{y}{x}}=x-\dfrac{y}{\dfrac{x}{y}+\dfrac{y}{x}}\cdot\dfrac{xy}{xy}=x-\dfrac{xy^2}{x^2+y^2}=\dfrac{x\left(x^2+y^2\right)}{x^2+y^2}-\dfrac{xy^2}{x^2+y^2}=\dfrac{x^3+xy^2-xy^2}{x^2+y^2}=\dfrac{x^3}{x^2+y^2}$

69. $\dfrac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}}=\dfrac{\dfrac{1}{x^2}-\dfrac{1}{y^2}}{\dfrac{1}{x}+\dfrac{1}{y}}=\dfrac{\dfrac{y^2}{x^2y^2}-\dfrac{x^2}{x^2y^2}}{\dfrac{y}{xy}+\dfrac{x}{xy}}=\dfrac{y^2-x^2}{x^2y^2}\cdot\dfrac{xy}{y+x}=\dfrac{(y-x)(y+x)xy}{x^2y^2(y+x)}=\dfrac{y-x}{xy}$

Alternatively, $\dfrac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}}=\dfrac{\left(\dfrac{1}{x^2}-\dfrac{1}{y^2}\right)}{\left(\dfrac{1}{x}+\dfrac{1}{y}\right)}\cdot\dfrac{x^2y^2}{x^2y^2}=\dfrac{y^2-x^2}{xy^2+x^2y}=\dfrac{(y-x)(y+x)}{xy(y+x)}=\dfrac{y-x}{xy}$.

70. $\dfrac{x^{-1}+y^{-1}}{(x+y)^{-1}}=\dfrac{\dfrac{1}{x}+\dfrac{1}{y}}{\dfrac{1}{x+y}}=\dfrac{\dfrac{1}{x}+\dfrac{1}{y}}{\dfrac{1}{x+y}}\cdot\dfrac{xy(x+y)}{xy(x+y)}=\dfrac{y(x+y)+x(x+y)}{xy}$

$=\dfrac{xy+y^2+x^2+xy}{xy}=\dfrac{x^2+2xy+y^2}{xy}=\dfrac{(x+y)^2}{xy}$

71. $1-\dfrac{1}{1-\dfrac{1}{x}}=1-\dfrac{x}{x-1}=\dfrac{x-1-x}{x-1}=\dfrac{1}{1-x}$

72. $1+\dfrac{1}{1+\dfrac{1}{1+x}}=1+\dfrac{1+x}{(1+x)+1}=1+\dfrac{x+1}{x+2}=\dfrac{x+2+x+1}{x+2}=\dfrac{2x+3}{x+2}$

73. $\dfrac{\dfrac{1}{1+x+h}-\dfrac{1}{1+x}}{h}=\dfrac{(1+x)-(1+x+h)}{h(1+x)(1+x+h)}=-\dfrac{1}{(1+x)(1+x+h)}$

74. In calculus it is necessary to eliminate the h in the denominator, and we do this by rationalizing the numerator:

$$\frac{\dfrac{1}{\sqrt{x+h}}-\dfrac{1}{\sqrt{x}}}{h}=\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}=\frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}=-\frac{1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}.$$

75. $\dfrac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h}=\dfrac{x^2-(x+h)^2}{hx^2(x+h)^2}=\dfrac{x^2-\left(x^2+2xh+h^2\right)}{hx^2(x+h)^2}=-\dfrac{2x+h}{x^2(x+h)^2}$

76. $\dfrac{(x+h)^3-7(x+h)-\left(x^3-7x\right)}{h}=\dfrac{x^3+3x^2h+3xh^2+h^3-7x-7h-x^3+7x}{h}=\dfrac{3x^2h+3xh^2+h^3-7h}{h}$

$=\dfrac{h\left(3x^2+3xh+h^2-7\right)}{h}=3x^2+3xh+h^2-7$

77. $\sqrt{1+\left(\dfrac{x}{\sqrt{1-x^2}}\right)^2}=\sqrt{1+\dfrac{x^2}{1-x^2}}=\sqrt{\dfrac{1-x^2}{1-x^2}+\dfrac{x^2}{1-x^2}}=\sqrt{\dfrac{1}{1-x^2}}=\dfrac{1}{\sqrt{1-x^2}}$

78. $\sqrt{1+\left(x^3-\dfrac{1}{4x^3}\right)^2}=\sqrt{1+x^6-\dfrac{2x^3}{4x^3}+\dfrac{1}{16x^6}}=\sqrt{1+x^6-\tfrac{1}{2}+\dfrac{1}{16x^6}}=\sqrt{x^6+\tfrac{1}{2}+\dfrac{1}{16x^6}}$

$=\sqrt{\left(x^3+\dfrac{1}{4x^3}\right)^2}=\left|x^3+\dfrac{1}{4x^3}\right|$

79. $\dfrac{3(x+2)^2(x-3)^2-(x+2)^3(2)(x-3)}{(x-3)^4}=\dfrac{(x+2)^2(x-3)[3(x-3)-(x+2)(2)]}{(x-3)^4}$

$=\dfrac{(x+2)^2(3x-9-2x-4)}{(x-3)^3}=\dfrac{(x+2)^2(x-13)}{(x-3)^3}$

80. $\dfrac{2x(x+6)^4-x^2(4)(x+6)^3}{(x+6)^8}=\dfrac{(x+6)^3\left[2x(x+6)-4x^2\right]}{(x+6)^8}=\dfrac{2x^2+12x-4x^2}{(x+6)^5}=\dfrac{12x-2x^2}{(x+6)^5}=\dfrac{2x(6-x)}{(x+6)^5}$

81. $\dfrac{2(1+x)^{1/2}-x(1+x)^{-1/2}}{1+x}=\dfrac{(1+x)^{-1/2}[2(1+x)-x]}{1+x}=\dfrac{x+2}{(1+x)^{3/2}}$

82. $\dfrac{\left(1-x^2\right)^{1/2}+x^2\left(1-x^2\right)^{-1/2}}{1-x^2}=\dfrac{\left(1-x^2\right)^{-1/2}\left(1-x^2+x^2\right)}{1-x^2}=\dfrac{1}{\left(1-x^2\right)^{3/2}}$

83. $\dfrac{3(1+x)^{1/3}-x(1+x)^{-2/3}}{(1+x)^{2/3}}=\dfrac{(1+x)^{-2/3}[3(1+x)-x]}{(1+x)^{2/3}}=\dfrac{2x+3}{(1+x)^{4/3}}$

84. $\dfrac{(7-3x)^{1/2}+\frac{3}{2}x(7-3x)^{-1/2}}{7-3x}=\dfrac{(7-3x)^{-1/2}\left(7-3x+\frac{3}{2}x\right)}{7-3x}=\dfrac{7-\frac{3}{2}x}{(7-3x)^{3/2}}$

85. $\dfrac{1}{5-\sqrt{3}}=\dfrac{1}{5-\sqrt{3}}\cdot\dfrac{5+\sqrt{3}}{5+\sqrt{3}}=\dfrac{5+\sqrt{3}}{25-3}=\dfrac{5+\sqrt{3}}{22}$

86. $\dfrac{3}{2-\sqrt{5}}=\dfrac{\left(2+\sqrt{5}\right)3}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}=\dfrac{6+3\sqrt{5}}{4-5}=-6-3\sqrt{5}$

87. $\dfrac{2}{\sqrt{2}+\sqrt{7}}=\dfrac{2}{\sqrt{2}+\sqrt{7}}\cdot\dfrac{\sqrt{2}-\sqrt{7}}{\sqrt{2}-\sqrt{7}}=\dfrac{2\left(\sqrt{2}-\sqrt{7}\right)}{2-7}=\dfrac{2\left(\sqrt{2}-\sqrt{7}\right)}{-5}=\dfrac{2\left(\sqrt{7}-\sqrt{2}\right)}{5}$

88. $\dfrac{1}{\sqrt{x}+1}=\dfrac{1}{\sqrt{x}+1}\cdot\dfrac{\sqrt{x}-1}{\sqrt{x}-1}=\dfrac{\sqrt{x}-1}{x-1}$

89. $\dfrac{y}{\sqrt{3}+\sqrt{y}}=\dfrac{y}{\sqrt{3}+\sqrt{y}}\cdot\dfrac{\sqrt{3}-\sqrt{y}}{\sqrt{3}-\sqrt{y}}=\dfrac{y\left(\sqrt{3}-\sqrt{y}\right)}{3-y}=\dfrac{y\sqrt{3}-y\sqrt{y}}{3-y}$

90. $\dfrac{2(x-y)}{\sqrt{x}-\sqrt{y}}=\dfrac{2(x-y)}{\sqrt{x}-\sqrt{y}}\cdot\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}=\dfrac{2(x-y)\left(\sqrt{x}+\sqrt{y}\right)}{x-y}=2\left(\sqrt{x}+\sqrt{y}\right)=2\sqrt{x}+2\sqrt{y}$

91. $\dfrac{1-\sqrt{5}}{3}=\dfrac{1-\sqrt{5}}{3}\cdot\dfrac{1+\sqrt{5}}{1+\sqrt{5}}=\dfrac{1-5}{3\left(1+\sqrt{5}\right)}=\dfrac{-4}{3\left(1+\sqrt{5}\right)}$

92. $\dfrac{\sqrt{3}+\sqrt{5}}{2}=\dfrac{\sqrt{3}+\sqrt{5}}{2}\cdot\dfrac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}=\dfrac{3-5}{2\left(\sqrt{3}-\sqrt{5}\right)}=\dfrac{-2}{2\left(\sqrt{3}-\sqrt{5}\right)}=\dfrac{-1}{\sqrt{3}-\sqrt{5}}$

93. $\dfrac{\sqrt{r}+\sqrt{2}}{5}=\dfrac{\sqrt{r}+\sqrt{2}}{5}\cdot\dfrac{\sqrt{r}-\sqrt{2}}{\sqrt{r}-\sqrt{2}}=\dfrac{r-2}{5\left(\sqrt{r}-\sqrt{2}\right)}$

94. $$\begin{aligned}\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}&=\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}=\frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}\\&=\frac{-h}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}=\frac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}\end{aligned}$$

95. $\sqrt{x^2+1}-x=\dfrac{\sqrt{x^2+1}-x}{1}\cdot\dfrac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}=\dfrac{x^2+1-x^2}{\sqrt{x^2+1}+x}=\dfrac{1}{\sqrt{x^2+1}+x}$

96. $\sqrt{x+1}-\sqrt{x}=\dfrac{\sqrt{x+1}-\sqrt{x}}{1}\cdot\dfrac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}=\dfrac{x+1-x}{\sqrt{x+1}+\sqrt{x}}=\dfrac{1}{\sqrt{x+1}+\sqrt{x}}$

97. **(a)** $R=\dfrac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}}=\dfrac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}}\cdot\dfrac{R_1R_2}{R_1R_2}=\dfrac{R_1R_2}{R_2+R_1}$

(b) Substituting $R_1=10$ ohms and $R_2=20$ ohms gives $R=\dfrac{(10)(20)}{(20)+(10)}=\dfrac{200}{30}\approx 6.7$ ohms.

98. **(a)** The average cost $A=\dfrac{\text{Cost}}{\text{number of shirts}}=\dfrac{500+6x+0.01x^2}{x}$.

(b)

x	10	20	50	100	200	500	1000
Average cost	\$56.10	\$31.20	\$16.50	\$12.00	\$10.50	\$12.00	\$16.50

99.

x	2.80	2.90	2.95	2.99	2.999	3	3.001	3.01	3.05	3.10	3.20
$\dfrac{x^2-9}{x-3}$	5.80	5.90	5.95	5.99	5.999	?	6.001	6.01	6.05	6.10	6.20

From the table, we see that the expression $\dfrac{x^2-9}{x-3}$ approaches 6 as x approaches 3. We simplify the expression: $\dfrac{x^2-9}{x-3}=\dfrac{(x-3)(x+3)}{x-3}=x+3$, $x\neq 3$. Clearly as x approaches 3, $x+3$ approaches 6. This explains the result in the table.

100. No, squaring $\dfrac{2}{\sqrt{x}}$ changes its value by a factor of $\dfrac{2}{\sqrt{x}}$.

101. Answers will vary.

Algebraic Error	Counterexample
$\frac{1}{a}+\frac{1}{b}\neq\frac{1}{a+b}$	$\frac{1}{2}+\frac{1}{2}\neq\frac{1}{2+2}$
$(a+b)^2\neq a^2+b^2$	$(1+3)^2\neq 1^2+3^2$
$\sqrt{a^2+b^2}\neq a+b$	$\sqrt{5^2+12^2}\neq 5+12$
$\frac{a+b}{a}\neq b$	$\frac{2+6}{2}\neq 6$
$\frac{a}{a+b}\neq\frac{1}{b}$	$\frac{1}{1+1}\neq 1$
$\frac{a^m}{a^n}\neq a^{m/n}$	$\frac{3^5}{3^2}\neq 3^{5/2}$

102. (a) $\frac{5+a}{5}=\frac{5}{5}+\frac{a}{5}=1+\frac{a}{5}$, so the statement is true.

(b) This statement is false. For example, take $x=5$ and $y=2$. Then LHS $=\frac{x+1}{y+1}=\frac{5+1}{2+1}=\frac{6}{3}=2$, while RHS $=\frac{x}{y}=\frac{5}{2}$, and $2\neq\frac{5}{2}$.

(c) This statement is false. For example, take $x=0$ and $y=1$. Then LHS $=\frac{x}{x+y}=\frac{0}{0+1}=0$, while RHS $=\frac{1}{1+y}=\frac{1}{1+1}=\frac{1}{2}$, and $0\neq\frac{1}{2}$.

(d) This statement is false. For example, take $x=1$ and $y=1$. Then LHS $=2\left(\frac{a}{b}\right)=2\left(\frac{1}{1}\right)=2$, while RHS $=\frac{2a}{2b}=\frac{2}{2}=1$, and $2\neq 1$.

(e) This statement is true: $\frac{-a}{b}=(-a)\left(\frac{1}{b}\right)=(-1)(a)\left(\frac{1}{b}\right)=(-1)\left(\frac{a}{b}\right)=-\frac{a}{b}$.

(f) This statement is false. For example, take $x=2$. Then LHS $=\frac{2}{4+x}=\frac{2}{4+2}=\frac{2}{6}=\frac{1}{3}$, while RHS $=\frac{1}{2}+\frac{2}{x}=\frac{1}{2}+\frac{2}{2}=\frac{3}{2}$, and $\frac{1}{3}\neq\frac{3}{2}$.

103. (a)

x	1	3	$\frac{1}{2}$	$\frac{9}{10}$	$\frac{99}{100}$	$\frac{999}{1000}$	$\frac{9999}{10{,}000}$
$x+\frac{1}{x}$	2	3.333	2.5	2.011	2.0001	2.000001	2.00000001

It appears that the smallest possible value of $x+\frac{1}{x}$ is 2.

(b) Because $x>0$, we can multiply both sides by x and preserve the inequality: $x+\frac{1}{x}\geq 2\Leftrightarrow x\left(x+\frac{1}{x}\right)\geq 2x\Leftrightarrow x^2+1\geq 2x\Leftrightarrow x^2-2x+1\geq 0\Leftrightarrow(x-1)^2\geq 0$. The last statement is true for all $x>0$, and because each step is reversible, we have shown that $x+\frac{1}{x}\geq 2$ for all $x>0$.

1.5 EQUATIONS

1. (a) Yes. If $a = b$, then $a + x = b + x$, and vice versa.

(b) Yes. If $a = b$, then $ma = mb$ for $m \neq 0$, and vice versa.

(c) No. For example, $-5 \neq 5$, but $(-5)^2 = 5^2 = 25$.

2. (a) Take positive and negative square roots of both sides: $(x+5)^2 = 64 \Leftrightarrow x + 5 = \pm 8$.

(b) Subtract 5 from both sides: $(x+5)^2 + 5 = 64 \Leftrightarrow (x+5)^2 = 59$.

(c) Subtract 2 from both sides: $x^2 + x = 2 \Leftrightarrow x^2 + x - 2 = 0$.

3. (a) To solve the equation $x^2 - 4x - 5 = 0$ by factoring, we write $x^2 - 4x - 5 = (x-5)(x+1) = 0$ if $x = 5$ or $x = -1$.

(b) To solve by completing the square, we write $x^2 - 4x - 5 = 0 \Leftrightarrow x^2 - 4x + 4 = 5 + 4 \Leftrightarrow (x-2)^2 = 9 \Leftrightarrow$ $(x-2) = \pm 3 \Leftrightarrow x = 5$ or $x = -1$.

(c) To solve using the Quadratic Formula, we write $x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-5)}}{2(1)} = \dfrac{4 \pm \sqrt{36}}{2} = 2 \pm 3 \Leftrightarrow x = 5$ or $x = -1$.

4. (a) The solutions of the equation $x^2(x-4) = 0$ are $x = 0$ and $x = 4$.

(b) To solve the equation $x^3 - 4x^2 = 0$ we *factor* the left-hand side: $x^2(x-4) = 0$, as above.

5. (a) Isolating the radical in $\sqrt{2x} + x = 0$, we obtain $\sqrt{2x} = -x$.

(b) Now square both sides: $\left(\sqrt{2x}\right)^2 = (-x)^2 \Rightarrow 2x = x^2$.

(c) Solving the resulting quadratic equation, we find $2x = x^2 \Rightarrow x^2 - 2x = x(x-2) = 0$, so the solutions are $x = 0$ and $x = 2$.

(d) We substitute these possible solutions into the original equation: $\sqrt{2 \cdot 0} + 0 = 0$, so $x = 0$ is a solution, but $\sqrt{2 \cdot 2} + 2 = 4 \neq 0$, so $x = 2$ is not a solution. The only real solution is $x = 0$.

6. The equation $(x+1)^2 - 5(x+1) + 6 = 0$ is of *quadratic* type. To solve the equation we set $W = x + 1$. The resulting quadratic equation is $W^2 - 5W + 6 = 0 \Rightarrow (W-3)(W-2) = 0 \Rightarrow W = 2$ or $W = 3 \Rightarrow x + 1 = 2$ or $x + 1 = 3 \Rightarrow$ $x = 1$ or $x = 2$. You can verify that these are both solutions to the original equation.

7. To eliminate the denominators in the equation $\dfrac{3}{x} + \dfrac{5}{x+2} = 2$, multiply each side by the lowest common denominator $x(x+2)$ to get the equivalent equation $3(x+2) + 5x = 2x(x+2)$.

8. To eliminate the square root in the equation $2x + 1 = \sqrt{x+1}$, *square* each side to get the equation $(2x+1)^2 = x + 1$. (But don't forget that squaring sometimes introduces extraneous solutions.)

9. (a) When $x = -2$, LHS $= 4(-2) + 7 = -8 + 7 = -1$ and RHS $= 9(-2) - 3 = -18 - 3 = -21$. Since LHS $\neq$ RHS, $x = -2$ is not a solution.

(b) When $x = 2$, LHS $= 4(-2) + 7 = 8 + 7 = 15$ and RHS $= 9(2) - 3 = 18 - 3 = 15$. Since LHS $=$ RHS, $x = 2$ is a solution.

10. (a) When $x = 2$, LHS $= 1 - [2 - (3 - (2))] = 1 - [2 - 1] = 1 - 1 = 0$ and RHS $= 4(2) - (6 + (2)) = 8 - 8 = 0$. Since LHS $=$ RHS, $x = 2$ is a solution.

(b) When $x = 4$ LHS $= 1 - [2 - (3 - (4))] = 1 - [2 - (-1)] = 1 - 3 = -2$ and RHS $= 4(4) - (6 + (4)) = 16 - 10 = 6$. Since LHS $\neq$ RHS, $x = 4$ is not a solution.

11. (a) When $x = 2$, LHS $= \dfrac{1}{2} - \dfrac{1}{2-4} = \dfrac{1}{2} - \dfrac{1}{-2} = \dfrac{1}{2} + \dfrac{1}{2} = 1$ and RHS $= 1$. Since LHS $=$ RHS, $x = 2$ is a solution.

(b) When $x = 4$ the expression $\dfrac{1}{4-4}$ is not defined, so $x = 4$ is not a solution.

12. **(a)** When $x = 4$, LHS $= \dfrac{4^{3/2}}{4-6} = \dfrac{2^3}{-2} = \dfrac{8}{-2} = -4$ and RHS $= (4) - 8 = -4$. Since LHS $=$ RHS, $x = 4$ is a solution.

(b) When $x = 8$, LHS $= \dfrac{8^{3/2}}{8-6} = \dfrac{\left(2^3\right)^{3/2}}{2} = \dfrac{2^{9/2}}{2} = 2^{7/2}$ and RHS $= (8) - 8 = 0$. Since LHS$\neq$ RHS, $x = 8$ is not a solution.

13. $5x - 6 = 14 \Leftrightarrow 5x - 6 + 6 = 14 + 6 \Leftrightarrow 5x = 20 \Leftrightarrow x = 4$

14. $3x + 4 = 7 \Leftrightarrow 3x = 3 \Leftrightarrow x = 1$

15. $\frac{1}{2}x - 8 = 1 \Leftrightarrow \frac{1}{2}x = 9 \Leftrightarrow x = 18$

16. $3 + \frac{1}{3}x = 5 \Leftrightarrow \frac{1}{3}x = 2 \Leftrightarrow x = 6$

17. $-x + 3 = 4x \Leftrightarrow -x + 3 + x = 4x + x \Leftrightarrow 3 = 5x \Leftrightarrow x = \frac{3}{5}$

18. $2x + 3 = 7 - 3x \Leftrightarrow 5x = 4 \Leftrightarrow x = \frac{4}{5}$

19. $\dfrac{x}{3} - 2 = \dfrac{5}{3}x + 7 \Leftrightarrow 3\left(\dfrac{x}{3} - 2\right) = 3\left(\dfrac{5}{3}x + 7\right) \Leftrightarrow x - 6 = 5x + 21 \Leftrightarrow -27 = 4x \Leftrightarrow x = -\frac{27}{4}$

20. $\frac{2}{5}x - 1 = \frac{3}{10}x + 3 \Leftrightarrow 4x - 10 = 3x + 30 \Leftrightarrow x = 40$

21. $2(1 - x) = 3(1 + 2x) + 5 \Leftrightarrow 2 - 2x = 3 + 6x + 5 \Leftrightarrow 2 - 2x = 8 + 6x \Leftrightarrow -6 = 8x \Leftrightarrow x = -\frac{3}{4}$

22. $\frac{2}{3}y + \frac{1}{2}(y - 3) = \dfrac{y+1}{4} \Leftrightarrow 8y + 6(y - 3) = 3(y + 1) \Leftrightarrow 8y + 6y - 18 = 3y + 3 \Leftrightarrow 14y - 18 = 3y + 3 \Leftrightarrow 11y = 21 \Leftrightarrow y = \frac{21}{11}$

23. $x - \frac{1}{3}x - \frac{1}{2}x - 5 = 0 \Leftrightarrow 6x - 2x - 3x - 30 = 0$ (multiply both sides by the LCD, 6) $\Leftrightarrow x = 30$

24. $2x - \dfrac{x}{2} + \dfrac{x+1}{4} = 6x \Leftrightarrow 8x - 2x + x + 1 = 24x \Leftrightarrow 7x + 1 = 24x \Leftrightarrow 1 = 17x \Leftrightarrow x = \frac{1}{17}$

25. $\dfrac{1}{x} = \dfrac{4}{3x} + 1 \Rightarrow 3 = 4 + 3x$ (multiply both sides by the LCD, $3x$) $\Leftrightarrow -1 = 3x \Leftrightarrow x = -\frac{1}{3}$

26. $\dfrac{2x-1}{x+2} = \frac{4}{5} \Rightarrow 5(2x - 1) = 4(x + 2) \Leftrightarrow 10x - 5 = 4x + 8 \Leftrightarrow 6x = 13 \Leftrightarrow x = \frac{13}{6}$

27. $\dfrac{3}{x+1} - \frac{1}{2} = \dfrac{1}{3x+3} \Rightarrow 3(6) - (3x + 3) = 2$ [multiply both sides by the LCD, $6(x + 1)$] $\Leftrightarrow 18 - 3x - 3 = 2 \Leftrightarrow -3x + 15 = 2 \Leftrightarrow -3x = -13 \Leftrightarrow x = \frac{13}{3}$

28. $\dfrac{4}{x-1} + \dfrac{2}{x+1} = \dfrac{35}{x^2-1} \Rightarrow 4(x + 1) + 2(x - 1) = 35 \Leftrightarrow 4x + 4 + 2x - 2 = 35 \Leftrightarrow 6x + 2 = 35 \Leftrightarrow 6x = 33 \Leftrightarrow x = \frac{11}{2}$

29. $(t - 4)^2 = (t + 4)^2 + 32 \Leftrightarrow t^2 - 8t + 16 = t^2 + 8t + 16 + 32 \Leftrightarrow -16t = 32 \Leftrightarrow t = -2$

30. $\sqrt{3}x + \sqrt{12} = \dfrac{x+5}{\sqrt{3}} \Leftrightarrow 3x + 6 = x + 5$ (multiply both sides by $\sqrt{3}$) $\Leftrightarrow 2x = -1 \Leftrightarrow x = -\frac{1}{2}$

31. $PV = nRT \Leftrightarrow R = \dfrac{PV}{nT}$

32. $F = G\dfrac{mM}{r^2} \Leftrightarrow m = \dfrac{Fr^2}{GM}$

33. $P = 2l + 2w \Leftrightarrow 2w = P - 2l \Leftrightarrow w = \dfrac{P - 2l}{2}$

34. $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} \Leftrightarrow R_1R_2 = RR_2 + RR_1$ (multiply both sides by the LCD, RR_1R_2). Thus $R_1R_2 - RR_1 = RR_2 \Leftrightarrow R_1(R_2 - R) = RR_2 \Leftrightarrow R_1 = \dfrac{RR_2}{R_2 - R}$

35. $\dfrac{ax+b}{cx+d} = 2 \Leftrightarrow ax + b = 2(cx + d) \Leftrightarrow ax + b = 2cx + 2d \Leftrightarrow ax - 2cx = 2d - b \Leftrightarrow (a - 2c)x = 2d - b \Leftrightarrow x = \dfrac{2d - b}{a - 2c}$

36. $a-2[b-3(c-x)]=6 \Leftrightarrow a-2(b-3c+3x)=6 \Leftrightarrow a-2b+6c-6x=6 \Leftrightarrow -6x=6-a+2b-6c \Leftrightarrow$ $x=\dfrac{6-a+2b-6c}{-6}=-\dfrac{6-a+2b-6c}{6}$

37. $a^2x+(a-1)=(a+1)x \Leftrightarrow a^2x-(a+1)x=-(a-1) \Leftrightarrow \left(a^2-(a+1)\right)x=-a+1 \Leftrightarrow \left(a^2-a-1\right)x=-a+1$ $\Leftrightarrow x=\dfrac{-a+1}{a^2-a-1}$

38. $\dfrac{a+1}{b}=\dfrac{a-1}{b}+\dfrac{b+1}{a} \Leftrightarrow a(a+1)=a(a-1)+b(b+1) \Leftrightarrow a^2+a=a^2-a+b^2+b \Leftrightarrow 2a=b^2+b \Leftrightarrow$ $a=\frac{1}{2}\left(b^2+b\right)$

39. $V=\frac{1}{3}\pi r^2h \Leftrightarrow r^2=\dfrac{3V}{\pi h} \Rightarrow r=\pm\sqrt{\dfrac{3V}{\pi h}}$

40. $F=G\dfrac{mM}{r^2} \Leftrightarrow r^2=G\dfrac{mM}{F} \Rightarrow r=\pm\sqrt{G\dfrac{mM}{F}}$

41. $a^2+b^2=c^2 \Leftrightarrow b^2=c^2-a^2 \Rightarrow b=\pm\sqrt{c^2-a^2}$

42. $A=P\left(1+\dfrac{i}{100}\right)^2 \Leftrightarrow \dfrac{A}{P}=\left(1+\dfrac{i}{100}\right)^2 \Rightarrow 1+\dfrac{i}{100}=\pm\sqrt{\dfrac{A}{P}} \Leftrightarrow \dfrac{i}{100}=-1\pm\sqrt{\dfrac{A}{P}} \Leftrightarrow i=-100\pm 100\sqrt{\dfrac{A}{P}}$

43. $h=\frac{1}{2}gt^2+v_0t \Leftrightarrow \frac{1}{2}gt^2+v_0t-h=0$. Using the Quadratic Formula,
$t=\dfrac{-(v_0)\pm\sqrt{(v_0)^2-4\left(\frac{1}{2}g\right)(-h)}}{2\left(\frac{1}{2}g\right)}=\dfrac{-v_0\pm\sqrt{v_0^2+2gh}}{g}$.

44. $S=\dfrac{n(n+1)}{2} \Leftrightarrow 2S=n^2+n \Leftrightarrow n^2+n-2S=0$. Using the Quadratic Formula,
$n=\dfrac{-1\pm\sqrt{(1)^2-4(1)(-2S)}}{2(1)}=\dfrac{-1\pm\sqrt{1+8S}}{2}$.

45. $x^2+x-12=0 \Leftrightarrow (x-3)(x+4)=0 \Leftrightarrow x-3=0$ or $x+4=0$. Thus, $x=3$ or $x=-4$.

46. $x^2+3x-4=0 \Leftrightarrow (x-1)(x+4)=0 \Leftrightarrow x-1=0$ or $x+4=0$. Thus, $x=1$ or $x=-4$.

47. $x^2-7x+12=0 \Leftrightarrow (x-4)(x-3)=0 \Leftrightarrow x-4=0$ or $x-3=0$. Thus, $x=4$ or $x=3$.

48. $x^2+8x+12=0 \Leftrightarrow (x+2)(x+6)=0 \Leftrightarrow x+2=0$ or $x+6=0$. Thus, $x=-2$ or $x=-6$.

49. $4x^2-4x-15=0 \Leftrightarrow (2x+3)(2x-5)=0 \Leftrightarrow 2x+3=0$ or $2x-5=0$. Thus, $x=-\frac{3}{2}$ or $x=\frac{5}{2}$.

50. $2y^2+7y+3=0 \Leftrightarrow (y+3)(2y+1)=0 \Leftrightarrow y+3=0$ or $2y+1=0$. Thus, $y=-3$ or $y=-\frac{1}{2}$.

51. $3x^2+5x=2 \Leftrightarrow 3x^2+5x-2=0 \Leftrightarrow (3x-1)(x+2)=0 \Leftrightarrow 3x-1=0$ or $x+2=0$. Thus, $x=\frac{1}{3}$ or $x=-2$.

52. $6x(x-1)=21-x \Leftrightarrow 6x^2-6x=21-x \Leftrightarrow 6x^2-5x-21=0 \Leftrightarrow (2x+3)(3x-7)=0 \Leftrightarrow 2x+3=0$ or $3x-7=0$. If $2x+3=0$, then $x=-\frac{3}{2}$; if $3x-7=0$, then $x=\frac{7}{3}$.

53. $2x^2=8 \Leftrightarrow x^2=4 \Leftrightarrow x=\pm 2$.

54. $3x^2-27=0 \Leftrightarrow x^2=9 \Leftrightarrow x=\pm 3$.

55. $(2x-5)^2=81 \Leftrightarrow 2x-5=\pm 9 \Leftrightarrow 2x=-4$ or $2x=14 \Leftrightarrow x=-2$ or $x=7$

56. $(5x+1)^2+3=10 \Leftrightarrow (5x+1)^2=7 \Leftrightarrow 5x+1=\pm\sqrt{7} \Leftrightarrow 5x=-1\pm\sqrt{7} \Leftrightarrow x=\dfrac{-1\pm\sqrt{7}}{5}$

57. $x^2+2x-5=0 \Leftrightarrow x^2+2x=5 \Leftrightarrow x^2+2x+1=5+1 \Leftrightarrow (x+1)^2=6 \Rightarrow x+1=\pm\sqrt{6} \Leftrightarrow \quad x=-1\pm\sqrt{6}$

58. $x^2-4x+2=0 \Leftrightarrow x^2-4x=-2 \Leftrightarrow x^2-4x+4=-2+4 \Leftrightarrow (x-2)^2=2 \Rightarrow x-2=\pm\sqrt{2} \Leftrightarrow x=2\pm\sqrt{2}$

59. $x^2-6x-11=0 \Leftrightarrow x^2-6x=11 \Leftrightarrow x^2-6x+9=11+9 \Leftrightarrow (x-3)^2=20 \Rightarrow x-3=\pm 2\sqrt{5} \Leftrightarrow x=3\pm 2\sqrt{5}$

60. $x^2+3x-\frac{7}{4}=0 \Leftrightarrow x^2+3x=\dfrac{7}{4} \Leftrightarrow x^2+3x+\frac{9}{4}=\frac{7}{4}+\frac{9}{4} \Leftrightarrow \left(x+\frac{3}{2}\right)^2=\frac{16}{4}=4 \Rightarrow x+\frac{3}{2}=\pm 2 \Leftrightarrow x=-\frac{3}{2}\pm 2 \Leftrightarrow$ $x=\frac{1}{2}$ or $x=-\frac{7}{2}$.

61. $2x^2+8x+1=0 \Leftrightarrow x^2+4x+\frac{1}{2}=0 \Leftrightarrow x^2+4x=-\frac{1}{2} \Leftrightarrow x^2+4x+4=-\frac{1}{2}+4 \Leftrightarrow (x+2)^2=\frac{7}{2} \Rightarrow x+2=\pm\sqrt{\frac{7}{2}}$ $\Leftrightarrow x=-2\pm\frac{\sqrt{14}}{2}$

62. $3x^2-6x-1=0 \Leftrightarrow x^2-2x-\frac{1}{3}=0 \Leftrightarrow x^2-2x=\frac{1}{3} \Leftrightarrow x^2-2x+1=\frac{1}{3}+1 \Leftrightarrow (x-1)^2=\frac{4}{3} \Rightarrow x-1=\pm\sqrt{\frac{4}{3}} \Leftrightarrow$ $x=1\pm\frac{2\sqrt{3}}{3}$

63. $4x^2-x=0 \Leftrightarrow x^2-\frac{1}{4}x=0 \Leftrightarrow x^2-\frac{1}{4}x+\frac{1}{64}=\frac{1}{64} \Leftrightarrow \left(x-\frac{1}{8}\right)^2=\frac{1}{64} \Rightarrow x-\frac{1}{8}=\pm\frac{1}{8} \Leftrightarrow x=\frac{1}{8}\pm\frac{1}{8}$, so $x=\frac{1}{8}-\frac{1}{8}=0$ or $x=\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$.

64. $x^2=\frac{3}{4}x-\frac{1}{8} \Leftrightarrow x^2-\dfrac{3}{4}x=-\frac{1}{8} \Leftrightarrow x^2-\frac{3}{4}x+\frac{9}{64}=-\frac{1}{8}+\frac{9}{64} \Leftrightarrow \left(x-\frac{3}{8}\right)^2=\frac{1}{64} \Rightarrow x-\frac{3}{8}=\pm\frac{1}{8} \Leftrightarrow x=\frac{3}{8}\pm\frac{1}{8}$, so $x=\frac{1}{2}$ or $x=\frac{1}{4}$.

65. $x^2-2x-15=0 \Leftrightarrow (x+3)(x-5)=0 \Leftrightarrow x+3=0$ or $x-5=0$.Thus, $x=-3$ or $x=5$.

66. $x^2+5x-6=0 \Leftrightarrow (x-1)(x+6)=0 \Leftrightarrow x-1=0$ or $x+6=0$. Thus, $x=1$ or $x=-6$.

67. $x^2-13x+42=0 \Leftrightarrow (x-6)(x-7)=0 \Leftrightarrow x-6=0$ or $x-7=0$. Thus, $x=6$ or $x=7$.

68. $x^2+10x-600=0 \Leftrightarrow (x+30)(x-20)=0 \Leftrightarrow x+30=0$ or $x-20=0$. Thus, $x=-30$ or $x=20$.

69. $2x^2+x-3=0 \Leftrightarrow (x-1)(2x+3)=0 \Leftrightarrow x-1=0$ or $2x+3=0$. If $x-1=0$, then $x=1$; if $2x+3=0$, then $x=-\frac{3}{2}$.

70. $3x^2+7x+4=0 \Leftrightarrow (3x+4)(x+1)=0 \Leftrightarrow 3x+4=0$ or $x+1=0$. Thus, $x=-\frac{4}{3}$ or $x=-1$.

71. $3x^2+6x-5=0 \Leftrightarrow x^2+2x-\frac{5}{3}=0 \Leftrightarrow x^2+2x=\frac{5}{3} \Leftrightarrow x^2+2x+1=\frac{5}{3}+1 \Leftrightarrow (x+1)^2=\frac{8}{3} \Rightarrow x+1=\pm\sqrt{\frac{8}{3}} \Leftrightarrow$ $x=-1\pm\frac{2\sqrt{6}}{3}$

72. $x^2-6x+1=0 \Rightarrow$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(1)}}{2(1)}=\frac{6\pm\sqrt{36-4}}{2}=\frac{6\pm\sqrt{32}}{2}=\frac{6\pm 4\sqrt{2}}{2}=3\pm 2\sqrt{2}$$

73. $9x^2+12x+4=0 \Leftrightarrow (3x+2)^2=0 \Leftrightarrow x=-\frac{2}{3}$

74. $4x^2-4x+1=0 \Leftrightarrow (2x-1)^2=0 \Leftrightarrow x=\frac{1}{2}$

75. $4x^2+16x-9=0 \Leftrightarrow (2x-1)(2x+9)=0 \Leftrightarrow 2x-1=0$ or $2x+9=0$. If $2x-1=0$, then $x=\frac{1}{2}$; if $2x+9=0$, then $x=-\frac{9}{2}$.

76. $0=x^2-4x+1=0 \Rightarrow$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}=\frac{4\pm\sqrt{16-4}}{2}=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm 2\sqrt{3}}{2}=2\pm\sqrt{3}$$

77. $7x^2-2x+4=0 \Rightarrow x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-(-2)\pm\sqrt{(-2)^2-4(7)(4)}}{2(7)}=\dfrac{2\pm\sqrt{4-112}}{-4}$, so there is no real solution.

78. $w^2=3(w-1) \Leftrightarrow w^2-3w+3=0 \Leftrightarrow w=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-(-3)\pm\sqrt{(-3)^2-4(1)(3)}}{2(1)}=\dfrac{3\pm\sqrt{9-12}}{2}$, so there is no real solution.

79. $10y^2 - 16y + 5 = 0 \Rightarrow$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(10)(5)}}{2(10)} = \frac{16 \pm \sqrt{256 - 200}}{20} = \frac{16 \pm \sqrt{56}}{20} = \frac{8 \pm \sqrt{14}}{10}$$

80. $25x^2 + 70x + 49 = 0 \Leftrightarrow (5x+7)^2 = 0 \Leftrightarrow 5x + 7 = 0 \Leftrightarrow 5x = -7 \Leftrightarrow x = -\frac{7}{5}$

81. $D = b^2 - 4ac = (-6)^2 - 4(1)(1) = 32$. Since D is positive, this equation has two real solutions.

82. $3x^2 = 6x - 9 \Leftrightarrow 3x^2 - 6x + 9 = 0$. $D = b^2 - 4ac = (-6)^2 - 4(3)(9) = 36 - 108 = -72$. Since D is negative, this equation has no real solution.

83. $D = b^2 - 4ac = (2.20)^2 - 4(1)(1.21) = 4.84 - 4.84 = 0$. Since $D = 0$, this equation has one real solution.

84. $D = b^2 - 4ac = (2.21)^2 - 4(1)(1.21) = 4.8841 - 4.84 = 0.0441$. Since $D \neq 0$, this equation has two real solutions.

85. $D = b^2 - 4ac = (5)^2 - 4(4)\left(\frac{13}{8}\right) = 25 - 26 = -1$. Since D is negative, this equation has no real solution.

86. $D = b^2 - 4ac = (r)^2 - 4(1)(-s) = r^2 + 4s$. Since D is positive, this equation has two real solutions.

87. $\dfrac{x^2}{x+100} = 50 \Rightarrow x^2 = 50(x+100) = 50x + 5000 \Leftrightarrow x^2 - 50x - 5000 = 0 \Leftrightarrow (x-100)(x+50) = 0 \Leftrightarrow x - 100 = 0$ or $x + 50 = 0$. Thus $x = 100$ or $x = -50$. The solutions are 100 and -50.

88. $\dfrac{1}{x-1} - \dfrac{2}{x^2} = 0 \Leftrightarrow x^2 - 2(x-1) = 0 \Leftrightarrow x^2 - 2x + 2 = 0 \Rightarrow$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2}.$$ Since the radicand is negative, there is no real solution.

89. $\dfrac{1}{x-1} + \dfrac{1}{x+2} = \frac{5}{4} \Leftrightarrow 4(x-1)(x+2)\left(\dfrac{1}{x-1} + \dfrac{1}{x+2}\right) = 4(x-1)(x+2)\left(\frac{5}{4}\right) \Leftrightarrow$ $4(x+2) + 4(x-1) = 5(x-1)(x+2) \Leftrightarrow 4x + 8 + 4x - 4 = 5x^2 + 5x - 10 \Leftrightarrow 5x^2 - 3x - 14 = 0 \Leftrightarrow$ $(5x+7)(x-2) = 0$. If $5x + 7 = 0$, then $x = -\frac{7}{5}$; if $x - 2 = 0$, then $x = 2$. The solutions are $-\frac{7}{5}$ and 2.

90. $\dfrac{x+5}{x-2} = \dfrac{5}{x+2} + \dfrac{28}{x^2-4} \Rightarrow (x+2)(x+5) = 5(x-2) + 28 \Leftrightarrow x^2 + 7x + 10 = 5x - 10 + 28 \Leftrightarrow x^2 + 2x - 8 = 0 \Leftrightarrow$ $(x-2)(x+4) = 0 \Leftrightarrow x - 2 = 0$ or $x + 4 = 0 \Leftrightarrow x = 2$ or $x = -4$. However, $x = 2$ is inadmissible since we can't divide by 0 in the original equation, so the only solution is -4.

91. $\dfrac{10}{x} - \dfrac{12}{x-3} + 4 = 0 \Leftrightarrow x(x-3)\left(\dfrac{10}{x} - \dfrac{12}{x-3} + 4\right) = 0 \Leftrightarrow (x-3)10 - 12x + 4x(x-3) = 0 \Leftrightarrow$ $10x - 30 - 12x + 4x^2 - 12x = 0 \Leftrightarrow 4x^2 - 14x - 30 = 0 \Leftrightarrow 2\left(2x^2 - 7x - 15\right) = 2(2x+3)(x-5) = 0 \Leftrightarrow 2x + 3 = 0$ or $x - 5 = 0$. If $2x + 3 = 0$, then $x = -\frac{3}{2}$; if $x - 5 = 0$, then $x = 5$.

92. $\dfrac{x}{2x+7} - \dfrac{x+1}{x+3} = 1 \Leftrightarrow x(x+3) - (x+1)(2x+7) = (2x+7)(x+3) \Leftrightarrow x^2 + 3x - 2x^2 - 9x - 7 = 2x^2 + 13x + 21$ $\Leftrightarrow 3x^2 + 19x + 28 = 0 \Leftrightarrow (3x+7)(x+4) = 0$. Thus either $3x + 7 = 0$, so $x = -\frac{7}{3}$, or $x = -4$. The solutions are $-\frac{7}{3}$ and -4.

93. $5 = \sqrt{4x-3} \Rightarrow 5^2 = \left(\sqrt{4x-3}\right)^2 \Leftrightarrow 25 = 4x - 3 \Leftrightarrow 4x = 28 \Leftrightarrow x = 7$

94. $\sqrt{8x-1} = 3 \Rightarrow 8x - 1 = 9 \Leftrightarrow 8x = 10 \Leftrightarrow x = \frac{5}{4}$

95. $\sqrt{2x-1} = \sqrt{3x-5} \Leftrightarrow 2x - 1 = 3x - 5 \Leftrightarrow x = 4$

96. $\sqrt{3+x} = \sqrt{x^2+1} \Leftrightarrow 3 + x = x^2 + 1 \Leftrightarrow x^2 - x - 2 = 0 \Leftrightarrow (x+1)(x-2) = 0 \Leftrightarrow x = -1$ or $x = 2$.

97. $\sqrt{2x+1} + 1 = x \Leftrightarrow \sqrt{2x+1} = x - 1 \Rightarrow 2x + 1 = (x-1)^2 \Leftrightarrow 2x + 1 = x^2 - 2x + 1 \Leftrightarrow 0 = x^2 - 4x = x(x-4)$. Potential solutions are $x = 0$ and $x - 4 \Leftrightarrow x = 4$. These are only potential solutions since squaring is not a reversible operation. We must check each potential solution in the original equation. Checking $x = 0$: $\sqrt{2(0)+1} + 1 \stackrel{?}{=} (0) \Leftrightarrow$ $\sqrt{1} + 1 \stackrel{?}{=} 0$ is false. Checking $x = 4$: $\sqrt{2(4)+1} + 1 \stackrel{?}{=} (4), \sqrt{9} + 1 \stackrel{?}{=} 4, 3 + 1 \stackrel{?}{=} 4$ is true. Thus, the only solution is $x = 4$.

98. $\sqrt{5-x}+1=x-2 \Leftrightarrow \sqrt{5-x}=x-3 \Rightarrow 5-x=(x-3)^2 \Leftrightarrow 5-x=x^2-6x+9 \Leftrightarrow 0=x^2-5x+4=(x-4)(x-1)$. Potential solutions are $x=4$ and $x=1$. We must check each potential solution in the original equation.

Checking $x=4$: $\sqrt{5-(4)}+1 \stackrel{?}{=} (4)-2, \sqrt{1}+1 \stackrel{?}{=} 4-2, 1+1 \stackrel{?}{=} 2$ is true.

Checking $x=1$: $\sqrt{5-(1)}+1 \stackrel{?}{=} (1)-2, \sqrt{4}+1 \stackrel{?}{=} -1, 2+1 \stackrel{?}{=} -1$ is false. The only solution is $x=4$.

99. $2x+\sqrt{x+1}=8 \Leftrightarrow \sqrt{x+1}=8-2x \Rightarrow x+1=(8-2x)^2 \Leftrightarrow x+1=64-32x+4x^2 \Leftrightarrow 0=4x^2-33x+63=(4x-21)(x-3)$. Potential solutions are $x=\frac{21}{4}$ and $x=3$. Substituting each of these solutions into the original equation, we see that $x=3$ is a solution, but $x=\frac{21}{4}$ is not. Thus 3 is the only solution.

100. $x-\sqrt{9-3x}=0 \Leftrightarrow x=\sqrt{9-3x} \Rightarrow x^2=9-3x \Leftrightarrow x^2+3x-9=0 \Leftrightarrow$ $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-3\pm\sqrt{3^2-4(1)(-9)}}{2(1)}=\dfrac{-3\pm\sqrt{9+36}}{2}=\dfrac{-3\pm3\sqrt{5}}{2}$. However, $\dfrac{-3-3\sqrt{5}}{2}$ is not a solution because $\dfrac{-3-3\sqrt{5}}{2}-\sqrt{9-3\left(\dfrac{-3-3\sqrt{5}}{2}\right)}$ is the sum of two negative nonzero numbers and hence is nonzero. Thus, the only solution is $x=\dfrac{3\sqrt{5}-3}{2}$.

101. $\sqrt{3x+1}=2+\sqrt{x+1} \Leftrightarrow 3x+1=\left(2+\sqrt{x+1}\right)^2=4+4\sqrt{x+1}+(x+1)=5+4\sqrt{x+1}+x \Leftrightarrow 2x-4=4\sqrt{x+1}$ $\Leftrightarrow x-2=2\sqrt{x+1} \Leftrightarrow x^2-4x+4=4(x+1) \Leftrightarrow x^2-8x=0 \Leftrightarrow x(x-8)=0 \Leftrightarrow x=0$ or 8. However, $x=0$ does not satisfy the original equation, so the only solution is $x=8$.

102. $\sqrt{1+x}+\sqrt{1-x}=2 \Rightarrow \left(\sqrt{1+x}+\sqrt{1-x}\right)^2=2^2 \Leftrightarrow 1+x+1-x+2\sqrt{(1+x)(1-x)}=4 \Leftrightarrow 2+2\sqrt{1-x^2}=4$ $\Leftrightarrow \sqrt{1-x^2}=1 \Rightarrow 1-x^2=1 \Rightarrow x=0$, which satisfies the original equation.

103. Let $w=x^2$. Then $x^4-13x^2+40=\left(x^2\right)^2-13x^2+40=0$ becomes $w^2-13w+40=0 \Leftrightarrow (w-5)(w-8)=0$. So $w-5=0 \Leftrightarrow w=5$, and $w-8=0 \Leftrightarrow w=8$. When $w=5$, we have $x^2=5 \Rightarrow x=\pm\sqrt{5}$. When $w=8$, we have $x^2=8 \Rightarrow x=\pm\sqrt{8}=\pm2\sqrt{2}$. The solutions are $\pm\sqrt{5}$ and $\pm2\sqrt{2}$.

104. $0=x^4-5x^2+4=\left(x^2-4\right)\left(x^2-1\right)=(x-2)(x+2)(x-1)(x+1)$. So $x=2$, $x=-2$, or $x=1$, or $x=-1$. The solutions are -2, 2, -1, and 1.

105. $2x^4+4x^2+1=0$. The LHS is the sum of two nonnegative numbers and a positive number, so $2x^4+4x^2+1\geq1>0$. This equation has no real solution.

106. $0=x^6-2x^3-3=\left(x^3-3\right)\left(x^3+1\right)$. Let $W=x^3$, so the equation becomes $(W-3)(W+1)=0$. If $W=3$, then $x^3=3 \Leftrightarrow x=\sqrt[3]{3}$, and if $W=1$, then $x^3=-1 \Leftrightarrow x=-1$. Thus $x=\sqrt[3]{3}$ or $x=-1$. The solutions are $\sqrt[3]{3}$ and -1.

107. Let $u=x^{2/3}$. Then $0=x^{4/3}-5x^{2/3}+6$ becomes $u^2-5u+6=0 \Leftrightarrow (u-3)(u-2)=0 \Leftrightarrow u-3=0$ or $u-2=0$. If $u-3=0$, then $x^{2/3}-3=0 \Leftrightarrow x^{2/3}=3 \Rightarrow x=\pm3^{3/2}$. If $u-2=0$, then $x^{2/3}-2=0 \Leftrightarrow x^{2/3}=2 \Rightarrow x=\pm2^{3/2}$. The solutions are $\pm3^{3/2}$ and $\pm2^{3/2}$; that is, $x=\pm2\sqrt{2}$ and $x=\pm3\sqrt{3}$.

108. Let $u=\sqrt[4]{x}$. Then $0=\sqrt{x}-3\sqrt[4]{x}-4=u^2-3u-4=(u-4)(u+1)$. So $u-4=\sqrt[4]{x}-4=0 \Leftrightarrow \sqrt[4]{x}=4 \Rightarrow x=4^4=256$, or $u+1=\sqrt[4]{x}+1=0 \Leftrightarrow \sqrt[4]{x}=-1$. However, $\sqrt[4]{x}$ is the positive fourth root, so this cannot equal -1. The only solution is 256.

109. Let $W=(x+1)^{1/2}$. Then $4(x+1)^{1/2}-5(x+1)^{3/2}+(x+1)^{5/2}=0 \Leftrightarrow 4W-5W^3+W^5=0 \Leftrightarrow W\left(W^4-5W^2+4\right)=0 \Leftrightarrow W\left(W^2-4\right)\left(W^2-1\right)=0 \Leftrightarrow W(W-2)(W+2)(W-1)(W+1)=0 \Leftrightarrow W=-2$, -1, 0, 1, or 2. If $W<0$, then $W=\sqrt{x+1}$ has no solution. If $W=0$, then $x=-1$; if $W=1$, then $\sqrt{x+1}=1 \Leftrightarrow x=1$, and if $W=2$, then $\sqrt{x+1}=2 \Leftrightarrow x=3$. The solutions are -1, 0, and 3.

110. $x^{1/2}+3x^{-1/2}=10x^{-3/2} \Leftrightarrow 0=x^{1/2}+3x^{-1/2}-10x^{-3/2}=x^{-3/2}\left(x^2+3x-10\right)=x^{-3/2}(x-2)(x+5)$. Now $x^{-3/2}$ never equals 0, and no solution can be negative, because we cannot take the square root of a negative number. Thus 2 is the only solution.

111. Let $u=x^{1/6}$. (We choose the exponent $\frac{1}{6}$ because the LCD of 2, 3, and 6 is 6.) Then $x^{1/2}-3x^{1/3}=3x^{1/6}-9 \Leftrightarrow x^{3/6}-3x^{2/6}=3x^{1/6}-9 \Leftrightarrow u^3-3u^2=3u-9 \Leftrightarrow 0=u^3-3u^2-3u+9=u^2(u-3)-3(u-3)=(u-3)\left(u^2-3\right)$. So $u-3=0$ or $u^2-3=0$. If $u-3=0$, then $x^{1/6}-3=0 \Leftrightarrow x^{1/6}=3 \Leftrightarrow x=3^6=729$. If $u^2-3=0$, then $x^{1/3}-3=0 \Leftrightarrow x^{1/3}=3 \Leftrightarrow x=3^3=27$. The solutions are 729 and 27.

112. Let $u=\sqrt{x}$. Then $0=x-5\sqrt{x}+6$ becomes $u^2-5u+6=(u-3)(u-2)=0$. If $u-3=0$, then $\sqrt{x}-3=0 \Leftrightarrow \sqrt{x}=3 \Rightarrow x=9$. If $u-2=0$, then $\sqrt{x}-2=0 \Leftrightarrow \sqrt{x}=2 \Rightarrow x=4$. The solutions are 9 and 4.

113. $|3x+5|=1$. So $3x+5=1 \Leftrightarrow 3x=-4 \Leftrightarrow x=-\frac{4}{3}$ or $3x+5=-1 \Leftrightarrow 3x=-6 \Leftrightarrow x=-2$.

114. $|2x|=3$. So $2x=3 \Leftrightarrow x=\frac{3}{2}$ or $2x=-3 \Leftrightarrow x=-\frac{3}{2}$.

115. $|x-4|=0.01$. So $x-4=0.01 \Leftrightarrow x=4.01$ or $x-4=-0.01 \Leftrightarrow x=3.99$.

116. $|x-6|=-1$. Since absolute value is never negative, this equation has no real solution.

117. $\dfrac{1}{x^3}+\dfrac{4}{x^2}+\dfrac{4}{x}=0 \Leftrightarrow 1+4x+4x^2=0 \Leftrightarrow (1+2x)^2=0 \Leftrightarrow 1+2x=0 \Leftrightarrow 2x=-1 \Leftrightarrow x=-\frac{1}{2}$. The solution is $-\frac{1}{2}$.

118. $0=4x^{-4}-16x^{-2}+4$. Multiplying by $\dfrac{x^4}{4}$ we get, $0=1-4x^2+x^4$. Substituting $u=x^2$, we get $0=1-4u+u^2$, and using the Quadratic Formula, we get $u=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}=\frac{4\pm\sqrt{16-4}}{2}=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm2\sqrt{3}}{2}=2\pm\sqrt{3}$. Substituting back, we have $x^2=2\pm\sqrt{3}$, and since $2+\sqrt{3}$ and $2-\sqrt{3}$ are both positive we have $x=\pm\sqrt{2+\sqrt{3}}$ or $x=\pm\sqrt{2-\sqrt{3}}$. Thus the solutions are $-\sqrt{2-\sqrt{3}}$, $\sqrt{2-\sqrt{3}}$, $-\sqrt{2+\sqrt{3}}$, and $\sqrt{2+\sqrt{3}}$.

119. $\sqrt{\sqrt{x+5}+x}=5$. Squaring both sides, we get $\sqrt{x+5}+x=25 \Leftrightarrow \sqrt{x+5}=25-x$. Squaring both sides again, we get $x+5=(25-x)^2 \Leftrightarrow x+5=625-50x+x^2 \Leftrightarrow 0=x^2-51x+620=(x-20)(x-31)$. Potential solutions are $x=20$ and $x=31$. We must check each potential solution in the original equation.

Checking $x=20$: $\sqrt{\sqrt{20+5}+20}=5 \Leftrightarrow \sqrt{\sqrt{25}+20}=5 \Leftrightarrow \sqrt{5+20}=5$, which is true, and hence $x=20$ is a solution.

Checking $x=31$: $\sqrt{\sqrt{(31)+5}+31}=5 \Leftrightarrow \sqrt{\sqrt{36}+31}=5 \Leftrightarrow \sqrt{37}=5$, which is false, and hence $x=31$ is not a solution. The only real solution is $x=20$.

120. $\sqrt[3]{4x^2-4x}=x \Leftrightarrow 4x^2-4x=x^3 \Leftrightarrow 0=x^3-4x^2+4x=x\left(x^2-4x+4\right)=x(x-2)^2$. So $x=0$ or $x=2$. The solutions are 0 and 2.

121. $x^2\sqrt{x+3}=(x+3)^{3/2} \Leftrightarrow 0=x^2\sqrt{x+3}-(x+3)^{3/2} \Leftrightarrow 0=\sqrt{x+3}\left[\left(x^2\right)-(x+3)\right] \Leftrightarrow 0=\sqrt{x+3}\left(x^2-x-3\right)$. If $(x+3)^{1/2}=0$, then $x+3=0 \Leftrightarrow x=-3$. If $x^2-x-3=0$, then by the Quadratic Formula, $x=\frac{1\pm\sqrt{13}}{2}$. The solutions are -3 and $\frac{1\pm\sqrt{13}}{2}$.

122. Let $u=\sqrt{11-x^2}$. By definition of u we require it to be nonnegative. Now $\sqrt{11-x^2}-\dfrac{2}{\sqrt{11-x^2}}=1 \Leftrightarrow u-\dfrac{2}{u}=1$. Multiplying both sides by u we obtain $u^2-2=u \Leftrightarrow 0=u^2-u-2=(u-2)(u+1)$. So $u=2$ or $u=-1$. But since u must be nonnegative, we only have $u=2 \Leftrightarrow \sqrt{11-x^2}=2 \Leftrightarrow 11-x^2=4 \Leftrightarrow x^2=7 \Leftrightarrow x=\pm\sqrt{7}$. The solutions are $\pm\sqrt{7}$.

123. $0=x^4-5ax^2+4a^2=\left(a-x^2\right)\left(4a-x^2\right)$. Since a is positive, $a-x^2=0 \Leftrightarrow x^2=a \Leftrightarrow x=\sqrt{a}$. Again, since a is positive, $4a-x^2=0 \Leftrightarrow x^2=4a \Leftrightarrow x=\pm2\sqrt{a}$. Thus the four solutions are $\pm\sqrt{a}$ and $\pm2\sqrt{a}$.

124. $0 = a^3x^3 + b^3 = (ax+b)\left(a^2x^2 - abx + b^2\right)$. So $ax + b = 0 \Leftrightarrow ax = -b \Leftrightarrow x = -\dfrac{b}{a}$ or

$x = \dfrac{-(-ab) \pm \sqrt{(-ab)^2 - 4\left(a^2\right)\left(b^2\right)}}{2\left(a^2\right)} = \dfrac{ab \pm \sqrt{-3a^2b^2}}{2a^2}$, but this gives no real solution. Thus, the solution is $x = -\dfrac{b}{a}$.

125. $\sqrt{x+a} + \sqrt{x-a} = \sqrt{2}\sqrt{x+6}$. Squaring both sides, we have
$x+a+2\left(\sqrt{x+a}\right)\left(\sqrt{x-a}\right)+x-a = 2(x+6) \Leftrightarrow 2x+2\left(\sqrt{x+a}\right)\left(\sqrt{x-a}\right) = 2x+12 \Leftrightarrow 2\left(\sqrt{x+a}\right)\left(\sqrt{x-a}\right) = 12$
$\Leftrightarrow \left(\sqrt{x+a}\right)\left(\sqrt{x-a}\right) = 6$. Squaring both sides again we have $(x+a)(x-a) = 36 \Leftrightarrow x^2 - a^2 = 36 \Leftrightarrow x^2 = a^2 + 36$
$\Leftrightarrow x = \pm\sqrt{a^2+36}$. Checking these answers, we see that $x = -\sqrt{a^2+36}$ is not a solution (for example, try substituting $a = 8$), but $x = \sqrt{a^2+36}$ is a solution.

126. Let $w = x^{1/6}$. Then $x^{1/3} = w^2$ and $x^{1/2} = w^3$, and so
$0 = w^3 - aw^2 + bw - ab = w^2(w-a) + b(w-a) = \left(w^2+b\right)(w-a) = \left(\sqrt[3]{x}+b\right)\left(\sqrt[6]{x}-a\right)$. So $\sqrt[6]{x} - a = 0 \Leftrightarrow$
$a = \sqrt[6]{x} \Leftrightarrow x = a^6$ is one solution. Setting the first factor equal to zero, we have $\sqrt[3]{x} + b = 0 \Leftrightarrow \sqrt[3]{x} = -b \Leftrightarrow x = -b^3$.
However, the original equation includes the term $b\sqrt[6]{x}$, and we cannot take the sixth root of a negative number, so this is not a solution. The only solution is $x = a^6$.

127. Using $h_0 = 288$, we solve $0 = -16t^2 + 288$, for $t \geq 0$. So $0 = -16t^2 + 288 \Leftrightarrow 16t^2 = 288 \Leftrightarrow t^2 = 18 \Rightarrow$
$t = \pm\sqrt{18} = \pm 3\sqrt{2}$. Thus it takes $3\sqrt{2} \approx 4.24$ seconds for the ball the hit the ground.

128. **(a)** Using $h_0 = 96$, half the distance is 48, so we solve the equation $48 = -16t^2 + 96 \Leftrightarrow -48 = -16t^2 \Leftrightarrow 3 = t^2 \Rightarrow$
$t = \pm\sqrt{3}$. Since $t \geq 0$, it takes $\sqrt{3} \approx 1.732$ s.

(b) The ball hits the ground when $h = 0$, so we solve the equation $0 = -16t^2 + 96 \Leftrightarrow 16t^2 = 96 \Leftrightarrow t^2 = 6 \Rightarrow t = \pm\sqrt{6}$.
Since $t \geq 0$, it takes $\sqrt{6} \approx 2.449$ s.

129. We are given $v_o = 40$ ft/s.

(a) Setting $h = 24$, we have $24 = -16t^2 + 40t \Leftrightarrow 16t^2 - 40t + 24 = 0 \Leftrightarrow 8\left(2t^2 - 5t + 3\right) = 0 \Leftrightarrow 8(2t-3)(t-1) = 0$
$\Leftrightarrow t = 1$ or $t = 1\frac{1}{2}$.Therefore, the ball reaches 24 feet in 1 second (ascending) and again after $1\frac{1}{2}$ seconds (descending).

(b) Setting $h = 48$, we have $48 = -16t^2 + 40t \Leftrightarrow 16t^2 - 40t + 48 = 0 \Leftrightarrow 2t^2 - 5t + 6 = 0 \Leftrightarrow$
$t = \dfrac{5 \pm \sqrt{25-48}}{4} = \dfrac{5 \pm \sqrt{-23}}{4}$. However, since the discriminant $D < 0$, there are no real solutions, and hence the ball never reaches a height of 48 feet.

(c) The greatest height h is reached only once. So $h = -16t^2 + 40t \Leftrightarrow 16t^2 - 40t + h = 0$ has only one solution. Thus
$D = (-40)^2 - 4(16)(h) = 0 \Leftrightarrow 1600 - 64h = 0 \Leftrightarrow h = 25$. So the greatest height reached by the ball is 25 feet.

(d) Setting $h = 25$, we have $25 = -16t^2 + 40t \Leftrightarrow 16t^2 - 40t + 25 = 0 \Leftrightarrow (4t-5)^2 = 0 \Leftrightarrow t = 1\frac{1}{4}$. Thus the ball reaches the highest point of its path after $1\frac{1}{4}$ seconds.

(e) Setting $h = 0$ (ground level), we have $0 = -16t^2 + 40t \Leftrightarrow 2t^2 - 5t = 0 \Leftrightarrow t(2t-5) = 0 \Leftrightarrow t = 0$ (start) or $t = 2\frac{1}{2}$.
So the ball hits the ground in $2\frac{1}{2}$ s.

130. If the maximum height is 100 feet, then the discriminant of the equation, $16t^2 - v_o t + 100 = 0$, must equal zero. So
$0 = b^2 - 4ac = (-v_o)^2 - 4(16)(100) \Leftrightarrow v_o^2 = 6400 \Rightarrow v_o = \pm 80$. Since $v_o = -80$ does not make sense, we must have $v_o = 80$ ft/s.

131. **(a)** The shrinkage factor when $w = 250$ is $S = \dfrac{0.032(250) - 2.5}{10{,}000} = \dfrac{8 - 2.5}{10{,}000} = 0.00055$. So the beam shrinks
$0.00055 \times 12.025 \approx 0.007$ m, so when it dries it will be $12.025 - 0.007 = 12.018$ m long.

(b) Substituting $S = 0.00050$ we get $0.00050 = \dfrac{0.032w - 2.5}{10{,}000} \Leftrightarrow 5 = 0.032w - 2.5 \Leftrightarrow 7.5 = 0.032w \Leftrightarrow$ $w = \dfrac{7.5}{0.032} \approx 234.375$. So the water content should be 234.375 kg/m^3.

132. Let d be the distance from the lens to the object. Then the distance from the lens to the image is $d - 4$. So substituting $F = 4.8$, $x = d$, and $y = d - 4$, and then solving for x, we have $\dfrac{1}{4.8} = \dfrac{1}{d} + \dfrac{1}{d-4}$. Now we multiply by the LCD, $4.8d\,(d-4)$, to get $d\,(d-4) = 4.8\,(d-4) + 4.8d \Leftrightarrow d^2 - 4d = 9.6d - 19.2 \Leftrightarrow 0 = d^2 - 13.6d + 19.2 \Rightarrow$ $d = \dfrac{13.6 \pm 10.4}{2}$. So $d = 1.6$ or $d = 12$. Since $d - 4$ must also be positive, the object is 12 cm from the lens.

133. **(a)** The fish population on January 1, 2002 corresponds to $t = 0$, so $F = 1000\left(30 + 17\,(0) - (0)^2\right) = 30{,}000$. To find when the population will again reach this value, we set $F = 30{,}000$, giving

$30000 = 1000\left(30 + 17t - t^2\right) = 30000 + 17000t - 1000t^2 \Leftrightarrow 0 = 17000t - 1000t^2 = 1000t\,(17 - t) \Leftrightarrow t = 0$ or $t = 17$. Thus the fish population will again be the same 17 years later, that is, on January 1, 2019.

(b) Setting $F = 0$, we have $0 = 1000\left(30 + 17t - t^2\right) \Leftrightarrow t^2 - 17t - 30 = 0 \Leftrightarrow$

$t = \dfrac{17 \pm \sqrt{289 + 120}}{-2} = \dfrac{17 \pm \sqrt{409}}{-2} = \dfrac{17 \pm 20.22}{2}$. Thus $t \approx -1.612$ or $t \approx 18.612$.Since $t < 0$ is inadmissible, it follows that the fish in the lake will have died out 18.612 years after January 1, 2002, that is on August 12, 2020.

134. We want to solve for t when $P = 500$. Letting $u = \sqrt{t}$ and substituting, we have $500 = 3t + 10\sqrt{t} + 140 \Leftrightarrow$ $500 = 3u^2 + 10u + 140 \Leftrightarrow 0 = 3u^2 + 10u - 360 \Rightarrow u = \dfrac{-5 \pm \sqrt{1105}}{3}$. Since $u = \sqrt{t}$, we must have $u \geq 0$. So $\sqrt{t} = u = \dfrac{-5 + \sqrt{1105}}{3} \approx 9.414 \Rightarrow t =\approx 88.62$. So it will take 89 days for the fish population to reach 500.

135. Setting $P = 1250$ and solving for x, we have $1250 = \frac{1}{10}x\,(300 - x) = 30x - \frac{1}{10}x^2 \Leftrightarrow \frac{1}{10}x^2 - 30x + 1250 = 0$. Using the quadratic formula, $x = \dfrac{-(-30) \pm \sqrt{(-30)^2 - 4\left(\frac{1}{10}\right)(1250)}}{2\left(\frac{1}{10}\right)} = \dfrac{30 \pm \sqrt{900 - 500}}{0.2} = \dfrac{30 \pm 20}{0.2}$. Thus $x = \dfrac{30 - 20}{0.2} = 50$ or $x = \dfrac{30 + 20}{0.2} = 250$. Since he must have $0 \leq x \leq 200$, he should make 50 ovens per week.

136. Let x be the distance from the center of the earth to the dead spot (in thousands of miles). Now setting $F = 0$, we have $0 = -\dfrac{K}{x^2} + \dfrac{0.012K}{(239 - x)^2} \Leftrightarrow \dfrac{K}{x^2} = \dfrac{0.012K}{(239 - x)^2} \Leftrightarrow K\,(239 - x)^2 = 0.012Kx^2 \Leftrightarrow 57121 - 478x + x^2 = 0.012x^2 \Leftrightarrow$ $0.988x^2 - 478x + 57121 = 0$. Using the Quadratic Formula, we obtain

$$x = \frac{-(-478) \pm \sqrt{(-478)^2 - 4\,(0.988)\,(57{,}121)}}{2\,(0.988)} = \frac{478 \pm \sqrt{228{,}484 - 225{,}742.192}}{1.976}$$
$$= \frac{478 \pm \sqrt{2741.808}}{1.976} \approx \frac{478 \pm 52.362}{1.976} \approx 241.903 \pm 26.499.$$

So either $x \approx 241.903 + 26.499 \approx 268$ or $x \approx 241.903 - 26.499 \approx 215$. Since 268,000 is greater than the distance from the earth to the moon, we reject the first root; thus $x \approx 215{,}000$ miles.

137. Since the total time is 3 s, we have $3 = \dfrac{\sqrt{d}}{4} + \dfrac{d}{1090}$. Letting $w = \sqrt{d}$, we have $3 = \frac{1}{4}w + \frac{1}{1090}w^2 \Leftrightarrow \frac{1}{1090}w^2 + \frac{1}{4}w - 3 = 0$ $\Leftrightarrow 2w^2 + 545w - 6540 = 0 \Rightarrow w = \dfrac{-545 \pm 591.054}{4}$. Since $w \geq 0$, we have $\sqrt{d} = w \approx 11.51$, so $d = 132.56$. The well is 132.6 ft deep.

138. **(a)** $3(0)+k-5=k(0)-k+1 \Leftrightarrow k-5=-k+1 \Leftrightarrow 2k=6 \Leftrightarrow k=3$

(b) $3(1)+k-5=k(1)-k+1 \Leftrightarrow 3+k-5=k-k+1 \Leftrightarrow k-2=1 \Leftrightarrow k=3$

(c) $3(2)+k-5=k(2)-k+1 \Leftrightarrow 6+k-5=2k-k+1 \Leftrightarrow k+1=k+1$. Since both sides of this equation are equal, $x=2$ is a solution for every value of k. That is, $x=2$ is a solution to every member of this family of equations.

139. When we multiplied by x, we introduced $x=0$ as a solution. When we divided by $x-1$, we are really dividing by 0, since $x=1 \Leftrightarrow x-1=0$.

140. **(a)** $x^2-9x+20=0 \Leftrightarrow (x-4)(x-5)=0 \Leftrightarrow x=4$ or $x=5$. The product of the solutions is $4 \cdot 5=20$, the constant term in the original equation; and their sum is $4+5=9$, the negative of the coefficient of x in the original equation.

(b) $x^2-2x-8=(x+2)(x-4)=0 \Leftrightarrow x=-2$ or $x=4$. The product of the solutions is $-2(4)=-8=c$ and their sum is $-2+4=2=-b$.

$x^2+4x+2=0 \Leftrightarrow x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-4\pm\sqrt{4^2-4(1)(2)}}{2(1)}=\dfrac{-4\pm\sqrt{16-8}}{2}=-2\pm\sqrt{2}$. The product of the solutions is $\left(-2+\sqrt{2}\right)\left(-2-\sqrt{2}\right)=4-2=2=c$ and their sum is $\left(-2+\sqrt{2}\right)+\left(-2-\sqrt{2}\right)=-4=-b$.

(c) In general, the equation $x^2+bx+c=0$ has solutions $r_1=\dfrac{-b+\sqrt{b^2-4c}}{2}$ and $r_2=\dfrac{-b-\sqrt{b^2-4c}}{2}$.

Thus, $r_1r_2=\dfrac{-b+\sqrt{b^2-4c}}{2}\cdot\dfrac{-b-\sqrt{b^2-4c}}{2}=\dfrac{1}{4}\left[(-b)^2-\left(\sqrt{b^2-4c}\right)^2\right]=\frac{1}{4}\left(b^2-b^2+4c\right)=c$ and

$r_1+r_2=\dfrac{-b+\sqrt{b^2-4c}}{2}+\dfrac{-b-\sqrt{b^2-4c}}{2}=\dfrac{-2b}{2}=-b$.

141. **(a)** *Method 1:* Let $u=\sqrt{x}$, so $u^2=x$. Thus $x-\sqrt{x}-2=0$ becomes $u^2-u-2=0 \Leftrightarrow (u-2)(u+1)=0$. So $u=2$ or $u=-1$. If $u=2$, then $\sqrt{x}=2 \Rightarrow x=4$. If $u=-1$, then $\sqrt{x}=-1 \Rightarrow x=1$. So the possible solutions are 4 and 1. Checking $x=4$ we have $4-\sqrt{4}-2=4-2-2=0$. Checking $x=1$ we have $1-\sqrt{1}-2=1-1-2\neq 0$. The only solution is 4.

Method 2: $x-\sqrt{x}-2=0 \Leftrightarrow x-2=\sqrt{x} \Rightarrow x^2-4x+4=x \Leftrightarrow x^2-5x+4=0 \Leftrightarrow (x-4)(x-1)=0$. So the possible solutions are 4 and 1. Checking will result in the same solution.

(b) *Method 1:* Let $u=\dfrac{1}{x-3}$, so $u^2=\dfrac{1}{(x-3)^2}$. Thus $\dfrac{12}{(x-3)^2}+\dfrac{10}{x-3}+1=0$ becomes $12u^2+10u+1=0$. Using the quadratic formula, we have $u=\frac{-10\pm\sqrt{10^2-4(12)(1)}}{2(12)}=\frac{-10\pm\sqrt{52}}{24}=\frac{-10\pm2\sqrt{13}}{24}=\frac{-5\pm\sqrt{13}}{12}$. If $u=\frac{-5-\sqrt{13}}{12}$, then $\dfrac{1}{x-3}=\dfrac{-5-\sqrt{13}}{12} \Leftrightarrow x-3=\frac{12}{-5-\sqrt{13}}\cdot\frac{-5+\sqrt{13}}{-5+\sqrt{13}}=\frac{12\left(-5+\sqrt{13}\right)}{12}=-5+\sqrt{13}$. So $x=-2+\sqrt{13}$.

If $u=\frac{-5+\sqrt{13}}{12}$, then $\dfrac{1}{x-3}=\dfrac{-5+\sqrt{13}}{12} \Leftrightarrow x-3=\frac{12}{-5+\sqrt{13}}\cdot\frac{-5-\sqrt{13}}{-5-\sqrt{13}}=\frac{12\left(-5-\sqrt{13}\right)}{12}=-5-\sqrt{13}$. So $x=-2-\sqrt{13}$.

The solutions are $-2\pm\sqrt{13}$.

Method 2: Multiplying by the LCD, $(x-3)^2$, we get $(x-3)^2\left(\dfrac{12}{(x-3)^2}+\dfrac{10}{x-3}+1\right)=0\cdot(x-3)^2 \Leftrightarrow$

$12+10(x-3)+(x-3)^2=0 \Leftrightarrow 12+10x-30+x^2-6x+9=0 \Leftrightarrow x^2+4x-9=0$. Using the Quadratic Formula, we have $u=\frac{-4\pm\sqrt{4^2-4(1)(-9)}}{2}=\frac{-4\pm\sqrt{52}}{2}=\frac{-4\pm2\sqrt{13}}{22}=-2\pm\sqrt{13}$. The solutions are $-2\pm\sqrt{13}$.

1.6 COMPLEX NUMBERS

1. The imaginary number i has the property that $i^2 = -1$.

2. For the complex number $3 + 4i$ the real part is 3 and the imaginary part is 4.

3. **(a)** The complex conjugate of $3 + 4i$ is $\overline{3 + 4i} = 3 - 4i$.

(b) $(3 + 4i)\left(\overline{3 + 4i}\right) = 3^2 + 4^2 = 25$

4. If $3 + 4i$ is a solution of a quadratic equation with real coefficients, then $\overline{3 + 4i} = 3 - 4i$ is also a solution of the equation.

5. Yes, every real number a is a complex number of the form $a + 0i$.

6. Yes. For any complex number z, $z + \overline{z} = (a + bi) + \overline{(a + bi)} = a + bi + a - bi = 2a$, which is a real number.

7. $5 - 7i$: real part 5, imaginary part -7.

8. $-6 + 4i$: real part -6, imaginary part 4.

9. $\dfrac{-2 - 5i}{3} = -\frac{2}{3} - \frac{5}{3}i$: real part $-\frac{2}{3}$, imaginary part $-\frac{5}{3}$.

10. $\dfrac{4 + 7i}{2} = 2 + \frac{7}{2}i$: real part 2, imaginary part $\frac{7}{2}$.

11. 3: real part 3, imaginary part 0.

12. $-\frac{1}{2}$: real part $-\frac{1}{2}$, imaginary part 0.

13. $-\frac{2}{3}i$: real part 0, imaginary part $-\frac{2}{3}$.

14. $i\sqrt{3}$: real part 0, imaginary part $\sqrt{3}$.

15. $\sqrt{3} + \sqrt{-4} = \sqrt{3} + 2i$: real part $\sqrt{3}$, imaginary part 2.

16. $2 - \sqrt{-5} = 2 - i\sqrt{5}$: real part 2, imaginary part $-\sqrt{5}$.

17. $(3 + 2i) + 5i = 3 + (2 + 5)\,i = 3 + 7i$

18. $3i - (2 - 3i) = -2 + [3 - (-3)]\,i = -2 + 6i$

19. $(5 - 3i) + (-4 - 7i) = (5 - 4) + (-3 - 7)\,i = 1 - 10i$

20. $(-3 + 4i) - (2 - 5i) = (-3 - 2) + [4 - (-5)]\,i = -5 + 9i$

21. $(-6 + 6i) + (9 - i) = (-6 + 9) + (6 - 1)\,i = 3 + 5i$

22. $(3 - 2i) + \left(-5 - \frac{1}{3}i\right) = (3 - 5) + \left(-2 - \frac{1}{3}\right) i = -2 - \frac{7}{3}i$

23. $\left(7 - \frac{1}{2}i\right) - \left(5 + \frac{3}{2}i\right) = (7 - 5) + \left(-\frac{1}{2} - \frac{3}{2}\right) i = 2 - 2i$

24. $(-4 + i) - (2 - 5i) = -4 + i - 2 + 5i = (-4 - 2) + (1 + 5)\,i = -6 + 6i$

25. $(-12 + 8i) - (7 + 4i) = -12 + 8i - 7 - 4i = (-12 - 7) + (8 - 4)\,i = -19 + 4i$

26. $6i - (4 - i) = 6i - 4 + i = (-4) + (6 + 1)\,i = -4 + 7i$

27. $4(-1 + 2i) = -4 + 8i$

28. $-2(3 - 4i) = -6 + 8i$

29. $(7 - i)(4 + 2i) = 28 + 14i - 4i - 2i^2 = (28 + 2) + (14 - 4)\,i = 30 + 10i$

30. $(5 - 3i)(1 + i) = 5 + 5i - 3i - 3i^2 = (5 + 3) + (5 - 3)\,i = 8 + 2i$

31. $(6 + 5i)(2 - 3i) = 12 - 18i + 10i - 15i^2 = (12 + 15) + (-18 + 10)\,i = 27 - 8i$

32. $(-2 + i)(3 - 7i) = -6 + 14i + 3i - 7i^2 = (-6 + 7) + (14 + 3)\,i = 1 + 17i$

33. $(2 + 5i)(2 - 5i) = 2^2 - (5i)^2 = 4 - 25(-1) = 29$

34. $(3 - 7i)(3 + 7i) = 3^2 - (7i)^2 = 58$

35. $(2 + 5i)^2 = 2^2 + (5i)^2 + 2(2)(5i) = 4 - 25 + 20i = -21 + 20i$

36. $(3 - 7i)^2 = 3^2 + (7i)^2 - 2(3)(7i) = -40 - 42i$

37. $\dfrac{1}{i} = \dfrac{1}{i} \cdot \dfrac{i}{i} = \dfrac{i}{i^2} = \dfrac{i}{-1} = -i$

38. $\dfrac{1}{1 + i} = \dfrac{1}{1 + i} \cdot \dfrac{1 - i}{1 - i} = \dfrac{1 - i}{1 - i^2} = \dfrac{1 - i}{1 + 1} = \dfrac{1 - i}{2} = \frac{1}{2} - \frac{1}{2}i$

39. $\dfrac{2 - 3i}{1 - 2i} = \dfrac{2 - 3i}{1 - 2i} \cdot \dfrac{1 + 2i}{1 + 2i} = \dfrac{2 + 4i - 3i - 6i^2}{1 - 4i^2} = \dfrac{(2 + 6) + (4 - 3)\,i}{1 + 4} = \dfrac{8 + i}{5}$ or $\frac{8}{5} + \frac{1}{5}i$

40. $\dfrac{5 - i}{3 + 4i} = \dfrac{5 - i}{3 + 4i} \cdot \dfrac{3 - 4i}{3 - 4i} = \dfrac{15 - 20i - 3i + 4i^2}{9 - 16i^2} = \dfrac{(15 - 4) + (-20 - 3)\,i}{9 + 16} = \dfrac{11 - 23i}{25} = \frac{11}{25} - \frac{23}{25}i$

41. $\dfrac{10i}{1-2i} = \dfrac{10i}{1-2i} \cdot \dfrac{1+2i}{1+2i} = \dfrac{10i+20i^2}{1-4i^2} = \dfrac{-20+10i}{1+4} = \dfrac{5(-4+2i)}{5} = -4+2i$

42. $(2-3i)^{-1} = \dfrac{1}{2-3i} = \dfrac{1}{2-3i} \cdot \dfrac{2+3i}{2+3i} = \dfrac{2+3i}{4-9i^2} = \dfrac{2+3i}{4+9} = \dfrac{2+3i}{13} = \frac{2}{13} + \frac{3}{13}i$

43. $\dfrac{4+6i}{3i} = \dfrac{4+6i}{3i} \cdot \dfrac{3i}{3i} = \dfrac{12i+18i^2}{9i^2} = \dfrac{-18+12i}{-9} = \dfrac{-18}{-9} + \dfrac{12}{-9}i = 2 - \frac{4}{3}i$

44. $\dfrac{-3+5i}{15i} = \dfrac{-3+5i}{15i} \cdot \dfrac{15i}{15i} = \dfrac{-45i+75i^2}{225i^2} = \dfrac{-75-45i}{-225} = \dfrac{-75}{-225} + \dfrac{-45}{-225}i = \frac{1}{3} + \frac{1}{5}i$

45. $\dfrac{1}{1+i} - \dfrac{1}{1-i} = \dfrac{1}{1+i} \cdot \dfrac{1-i}{1-i} - \dfrac{1}{1-i} \cdot \dfrac{1+i}{1+i} = \dfrac{1-i}{1-i^2} - \dfrac{1+i}{1-i^2} = \dfrac{1-i}{2} + \dfrac{-1-i}{2} = -i$

46. $\dfrac{(1+2i)(3-i)}{2+i} = \dfrac{3-i+6i-2i^2}{2+i} = \dfrac{5+5i}{2+i} \cdot \dfrac{2-i}{2-i} = \dfrac{10-5i+10i-5i^2}{4-i^2} = \dfrac{(10+5)+(-5+10)i}{5}$
$= \dfrac{15+5i}{5} = \frac{15}{5} + \frac{5}{5}i = 3+i$

47. $i^3 = i^2 i = -i$

48. $i^{10} = \left(i^2\right)^5 = (-1)^5 = -1$

49. $(3i)^5 = 3^5\left(i^2\right)^2 i = 243(-1)^2 i = 243i$

50. $(2i)^4 = 2^4 i^4 = 16(1) = 16$

51. $i^{1000} = \left(i^4\right)^{250} = 1^{250} = 1$

52. $i^{1002} = \left(i^4\right)^{250} i^2 = 1i^2 = -1$

53. $\sqrt{-49} = \sqrt{49}\sqrt{-1} = 7i$

54. $\sqrt{\frac{-81}{16}} = \frac{9}{4}i$

55. $\sqrt{-3}\sqrt{-12} = i\sqrt{3} \cdot 2i\sqrt{3} = 6i^2 = -6$

56. $\sqrt{\frac{1}{3}}\sqrt{-27} = \sqrt{\frac{1}{3}} \cdot 3i\sqrt{3} = 3i$

57. $\left(3-\sqrt{-5}\right)\left(1+\sqrt{-1}\right) = \left(3-i\sqrt{5}\right)(1+i) = 3+3i-i\sqrt{5}-i^2\sqrt{5} = \left(3+\sqrt{5}\right) + \left(3-\sqrt{5}\right)i$

58. $\left(\sqrt{3}-\sqrt{-4}\right)\left(\sqrt{6}-\sqrt{-8}\right) = \left(\sqrt{3}-2i\right)\left(\sqrt{6}-2i\sqrt{2}\right) = \sqrt{18}-2i\sqrt{6}-2i\sqrt{6}+4i^2\sqrt{2}$
$= \left(3\sqrt{2}-4\sqrt{2}\right) + \left(-2\sqrt{6}-2\sqrt{6}\right)i = -\sqrt{2}-4i\sqrt{6}$

59. $\dfrac{2+\sqrt{-8}}{1+\sqrt{-2}} = \dfrac{2+2i\sqrt{2}}{1+i\sqrt{2}} = \dfrac{2\left(1+i\sqrt{2}\right)}{1+i\sqrt{2}} = 2$

60. $\dfrac{\sqrt{-36}}{\sqrt{-2}\sqrt{-9}} = \dfrac{6i}{i\sqrt{2} \cdot 3i} = \dfrac{2}{i\sqrt{2}} \cdot \dfrac{i\sqrt{2}}{i\sqrt{2}} = \dfrac{2i\sqrt{2}}{2i^2} = \dfrac{i\sqrt{2}}{-1} = -i\sqrt{2}$

61. $x^2+49 = 0 \Leftrightarrow x^2 = -49 \Rightarrow x = \pm 7i$

62. $3x^2+1 = 0 \Leftrightarrow 3x^2 = -1 \Leftrightarrow x^2 = -\frac{1}{3} \Leftrightarrow x = \pm\frac{\sqrt{3}}{3}i$

63. $x^2-x+2 = 0 \Leftrightarrow x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-(-1) \pm \sqrt{(-1)^2-4(1)(2)}}{2(1)} = \dfrac{1 \pm \sqrt{-7}}{2} = \frac{1}{2} \pm \frac{\sqrt{7}}{2}i$

64. $x^2+2x+2 = 0 \Rightarrow x = \dfrac{-2 \pm \sqrt{(2)^2-4(1)(2)}}{2(1)} = \dfrac{-2 \pm \sqrt{4-8}}{2} = \dfrac{-2 \pm \sqrt{-4}}{2} = \dfrac{-2 \pm 2i}{2} = -1 \pm i$

65. $x^2+3x+7 = 0 \Leftrightarrow x = \dfrac{-3 \pm \sqrt{3^2-4(1)(7)}}{2(1)} = \dfrac{-3 \pm \sqrt{-19}}{2} = -\frac{3}{2} \pm \frac{\sqrt{19}}{2}i$

66. $x^2-6x+10 = 0 \Rightarrow x = \dfrac{-(-6) \pm \sqrt{(-6)^2-4(1)(10)}}{2(1)} = \dfrac{6 \pm \sqrt{36-40}}{2} = \dfrac{6 \pm \sqrt{-4}}{2} = \dfrac{6 \pm 2i}{2} = 3 \pm i$

67. $x^2+x+1 = 0 \Rightarrow x = \dfrac{-(1) \pm \sqrt{(1)^2-4(1)(1)}}{2(1)} = \dfrac{-1 \pm \sqrt{1-4}}{2} = \dfrac{-1 \pm \sqrt{-3}}{2} = \dfrac{-1 \pm i\sqrt{3}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

68. $x^2 - 3x + 3 = 0 \Rightarrow x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(3)}}{2(1)} = \dfrac{3 \pm \sqrt{9-12}}{2} = \dfrac{3 \pm \sqrt{-3}}{2} = \dfrac{3 \pm i\sqrt{3}}{2} = \frac{3}{2} \pm \frac{\sqrt{3}}{2}i$

69. $2x^2 - 2x + 1 = 0 \Rightarrow x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(1)}}{2(2)} = \dfrac{2 \pm \sqrt{4-8}}{4} = \dfrac{2 \pm \sqrt{-4}}{4} = \dfrac{2 \pm 2i}{4} = \frac{1}{2} \pm \frac{1}{2}i$

70. $t + 3 + \dfrac{3}{t} = 0 \Leftrightarrow t^2 + 3t + 3 = 0 \Rightarrow t = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(3)}}{2(1)} = \frac{-3 \pm \sqrt{9-12}}{2} = \frac{-3 \pm \sqrt{-3}}{2} = \frac{-3 \pm i\sqrt{3}}{2} = -\frac{3}{2} \pm \frac{\sqrt{3}}{2}i$

71. $6x^2 + 12x + 7 = 0 \Rightarrow$

$x = \frac{-(12) \pm \sqrt{(12)^2 - 4(6)(7)}}{2(6)} = \frac{-12 \pm \sqrt{144-168}}{12} = \frac{-12 \pm \sqrt{-24}}{12} = \frac{-12 \pm 2i\sqrt{6}}{12} = \frac{-12}{12} \pm \frac{2i\sqrt{6}}{12} = -1 \pm \frac{\sqrt{6}}{6}i$

72. $x^2 + \frac{1}{2}x + 1 = 0 \Rightarrow$

$x = \dfrac{-\left(\frac{1}{2}\right) \pm \sqrt{\left(\frac{1}{2}\right)^2 - 4(1)(1)}}{2(1)} = \dfrac{-\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4}}{2} = \dfrac{-\frac{1}{2} \pm \sqrt{-\frac{15}{4}}}{2} = \dfrac{-\frac{1}{2} \pm \frac{1}{2}i\sqrt{15}}{2} = -\frac{1}{4} \pm \frac{\sqrt{15}}{4}i$

73. $\overline{z} + \overline{w} = \overline{3-4i} + \overline{5+2i} = 3 + 4i + 5 - 2i = 8 + 2i$

74. $\overline{z+w} = \overline{3 - 4i + 5 + 2i} = \overline{8-2i} = 8 + 2i$

75. $z \cdot \overline{z} = (3-4i)(3+4i) = 3^2 + 4^2 = 25$

76. $\overline{z} \cdot \overline{w} = (3+4i)(5-2i) = 15 - 6i + 20i - 8i^2 = 23 + 14i$

77. LHS $= \overline{z} + \overline{w} = \overline{(a+bi)} + \overline{(c+di)} = a - bi + c - di = (a+c) + (-b-d)i = (a+c) - (b+d)i$.

RHS $= \overline{z+w} = \overline{(a+bi)+(c+di)} = \overline{(a+c)+(b+d)i} = (a+c) - (b+d)i$.

Since LHS $=$ RHS, this proves the statement.

78. LHS $= \overline{zw} = \overline{(a+bi)(c+di)} = \overline{ac + adi + bci + bdi^2} = \overline{(ac-bd) + (ad+bc)i} = (ac-bd) - (ad+bc)i$.

RHS $= \overline{z} \cdot \overline{w} = \overline{a+bi} \cdot \overline{c+di} = (a-bi)(c-di) = ac - adi - bci + bdi^2 = (ac-bd) - (ad+bc)i$.

Since LHS $=$ RHS, this proves the statement.

79. LHS $= (\overline{z})^2 = \left(\overline{(a+bi)}\right)^2 = (a-bi)^2 = a^2 - 2abi + b^2i^2 = \left(a^2 - b^2\right) - 2abi$.

RHS $= \overline{z^2} = \overline{(a+bi)^2} = \overline{a^2 + 2abi + b^2i^2} = \overline{\left(a^2 - b^2\right) + 2abi} = \left(a^2 - b^2\right) - 2abi$.

Since LHS $=$ RHS, this proves the statement.

80. $\overline{\overline{z}} = \overline{\overline{a+bi}} = \overline{a-bi} = a + bi = z$.

81. $z + \overline{z} = (a+bi) + \overline{(a+bi)} = a + bi + a - bi = 2a$, which is a real number.

82. $z - \overline{z} = (a+bi) - \overline{(a+bi)} = a + bi - (a - bi) = a + bi - a + bi = 2bi$, which is a pure imaginary number.

83. $z \cdot \overline{z} = (a+bi) \cdot \overline{(a+bi)} = (a+bi) \cdot (a-bi) = a^2 - b^2i^2 = a^2 + b^2$, which is a real number.

84. Suppose $z = \overline{z}$. Then we have $(a+bi) = \overline{(a+bi)} \Rightarrow a + bi = a - bi \Rightarrow 0 = -2bi \Rightarrow b = 0$, so z is real. Now if z is real, then $z = a + 0i$ (where a is real). Since $\overline{z} = a - 0i$, we have $z = \overline{z}$.

85. Using the Quadratic Formula, the solutions to the equation are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Since both solutions are imaginary, we have $b^2 - 4ac < 0 \Leftrightarrow 4ac - b^2 > 0$, so the solutions are $x = \dfrac{-b}{2a} \pm \dfrac{\sqrt{4ac - b^2}}{2a}\,i$, where $\sqrt{4ac - b^2}$ is a real number. Thus the solutions are complex conjugates of each other.

86. $i = i$, $i^5 = i^4 \cdot i = i$, $i^9 = i^8 \cdot i = i$; $i^2 = -1$, $i^6 = i^4 \cdot i^2 = -1$, $i^{10} = i^8 \cdot i^2 = -1$;
$i^3 = -i$, $i^7 = i^4 \cdot i^3 = -i$, $i^{11} = i^8 \cdot i^3 = -i$; $i^4 = 1$, $i^8 = i^4 \cdot i^4 = 1$, $i^{12} = i^8 \cdot i^4 = 1$.

Because $i^4 = 1$, we have $i^n = i^r$, where r is the remainder when n is divided by 4, that is, $n = 4 \cdot k + r$, where k is an integer and $0 \le r < 4$. Since $4446 = 4 \cdot 1111 + 2$, we must have $i^{4446} = i^2 = -1$.

1.7 MODELING WITH EQUATIONS

1. An equation modeling a real-world situation can be used to help us understand a real-world problem using mathematical methods. We translate real-world ideas into the language of algebra to construct our model, and translate our mathematical results back into real-world ideas in order to interpret our findings.

2. In the formula $I = Prt$ for simple interest, P stands for *principal*, r for *interest rate*, and t for *time (in years)*.

3. **(a)** A square of side x has area $A = x^2$.

(b) A rectangle of length l and width w has area $A = lw$.

(c) A circle of radius r has area $A = \pi r^2$.

4. Balsamic vinegar contains 5% acetic acid, so a 32 ounce bottle of balsamic vinegar contains $32 \cdot 5\% = 32 \cdot \frac{5}{100} = 1.6$ ounces of acetic acid.

5. A painter paints a wall in x hours, so the fraction of the wall she paints in one hour is $\dfrac{1\text{ wall}}{x\text{ hours}} = \dfrac{1}{x}$.

6. Solving $d = rt$ for r, we find $\dfrac{d}{t} = \dfrac{rt}{t} \Rightarrow r = \dfrac{d}{t}$. Solving $d = rt$ for t, we find $\dfrac{d}{r} = \dfrac{rt}{r} \Rightarrow t = \dfrac{d}{r}$.

7. If n is the first integer, then $n + 1$ is the middle integer, and $n + 2$ is the third integer. So the sum of the three consecutive integers is $n + (n + 1) + (n + 2) = 3n + 3$.

8. If n is the middle integer, then $n - 1$ is the first integer, and $n + 1$ is the third integer. So the sum of the three consecutive integers is $(n - 1) + n + (n + 1) = 3n$.

9. If n is the first even integer, then $n + 2$ is the second even integer and $n + 4$ is the third. So the sum of three consecutive even integers is $n + (n + 2) + (n + 4) = 3n + 6$.

10. If n is the first integer, then the next integer is $n + 1$. The sum of their squares is
$n^2 + (n + 1)^2 = n^2 + \left(n^2 + 2n + 1\right) = 2n^2 + 2n + 1$.

11. If s is the third test score, then since the other test scores are 78 and 82, the average of the three test scores is $\dfrac{78 + 82 + s}{3} = \dfrac{160 + s}{3}$.

12. If q is the fourth quiz score, then since the other quiz scores are 8, 8, and 8, the average of the four quiz scores is $\dfrac{8 + 8 + 8 + q}{4} = \dfrac{24 + q}{4}$.

13. If x dollars are invested at $2\frac{1}{2}\%$ simple interest, then the first year you will receive $0.025x$ dollars in interest.

14. If n is the number of months the apartment is rented, and each month the rent is \$795, then the total rent paid is $795n$.

15. Since w is the width of the rectangle, the length is three times the width, or $4w$. Then
area $=$ length $\times$ width $= 4w \times w = 4w^2$ ft^2.

16. Since w is the width of the rectangle, the length is $w + 6$. The perimeter is

$$2 \times \text{length} + 2 \times \text{width} = 2(w + 6) + 2(w) = 4w + 12$$

17. If d is the given distance, in miles, and distance $=$ rate $\times$ time, we have time $= \dfrac{\text{distance}}{\text{rate}} = \dfrac{d}{55}$.

18. Since distance $=$ rate$\times$ time we have distance $= s \times (45\text{ min}) \dfrac{1\text{ h}}{60\text{ min}} = \frac{3}{4}s$ mi.

19. If x is the quantity of pure water added, the mixture will contain 25 oz of salt and $3 + x$ gallons of water. Thus the concentration is $\dfrac{25}{3 + x}$.

20. If p is the number of pennies in the purse, then the number of nickels is $2p$, the number of dimes is $4 + 2p$, and the number of quarters is $(2p) + (4 + 2p) = 4p + 4$. Thus the value (in cents) of the change in the purse is $1 \cdot p + 5 \cdot 2p + 10 \cdot (4 + 2p) + 25 \cdot (4p + 4) = p + 10p + 40 + 20p + 100p + 100 = 131p + 140$.

21. If d is the number of days and m the number of miles, then the cost of a rental is $C = 65d + 0.20m$. In this case, $d = 3$ and $C = 275$, so we solve for m: $275 = 65 \cdot 3 + 0.20m \Leftrightarrow 275 = 195 + 0.2m \Leftrightarrow 0.2m = 80 \Leftrightarrow m = \dfrac{80}{0.2} = 400$. Thus, Michael drove 400 miles.

22. If m is the number of messages, then a monthly cell phone bill (above \$10) is $B = 10 + 0.10\,(m - 1000)$. In this case, $B = 38.5$ and we solve for m: $38.5 = 10 + 0.10\,(m - 1000) \Leftrightarrow 0.10\,(m - 1000) = 28.5 \Leftrightarrow m - 1000 = \dfrac{28.5}{0.1} = 285 \Leftrightarrow m = 1285$. Thus, Miriam sent 1285 text messages in June.

23. If x is Linh's score on her final exam, then because the final counts twice as much as each midterm, her average score is $\dfrac{82 + 75 + 71 + 2x}{3\,(100) + 200} = \dfrac{228 + 2x}{500} = \dfrac{114 + x}{250}$. For her to average 80%, we must have $\dfrac{114 + x}{250} = 80\% = 0.8 \Leftrightarrow 114 + x = 250\,(0.8) = 200 \Leftrightarrow x = 86$. So Linh scored 86% on her final exam.

24. Six students scored 100 and three students scored 60. Let x be the average score of the remaining $25 - 6 - 3 = 16$ students. Because the overall average is $84\% = 0.84$, we have $\dfrac{6\,(100) + 3\,(60) + 16x}{25\,(100)} = 0.84 \Leftrightarrow 780 + 16x = 0.84\,(2500) = 2100 \Leftrightarrow 16x = 1320 \Leftrightarrow x = \frac{1320}{16} = 82.5$. Thus, the remaining 16 students' average score was 82.5%.

25. Let m be the amount invested at $4\frac{1}{2}\%$. Then $12{,}000 - m$ is the amount invested at 4%.

Since the total interest is equal to the interest earned at $4\frac{1}{2}\%$ plus the interest earned at 4%, we have $525 = 0.045m + 0.04\,(12{,}000 - m) \Leftrightarrow 525 = 0.045m + 480 - 0.04m \Leftrightarrow 45 = 0.005m \Leftrightarrow m = \dfrac{45}{0.005} = 9000$. Thus \$9000 is invested at $4\frac{1}{2}\%$, and $\$12{,}000 - 9000 = \$3000$ is invested at 4%.

26. Let m be the amount invested at $5\frac{1}{2}\%$. Then $4000 + m$ is the total amount invested. Thus

$$4\tfrac{1}{2}\% \text{ of the total investment} = \text{interest earned at } 4\% + \text{interest earned at } 5\tfrac{1}{2}\%$$

So $0.045\,(4000 + m) = 0.04\,(4000) + 0.055m \Leftrightarrow 180 + 0.045m = 160 + 0.055m \Leftrightarrow 20 = 0.01m \Leftrightarrow m = \dfrac{20}{0.01} = 2000$. Thus \$2,000 needs to be invested at $5\frac{1}{2}\%$.

27. Using the formula $I = Prt$ and solving for r, we get $262.50 = 3500 \cdot r \cdot 1 \Leftrightarrow r = \dfrac{262.5}{3500} = 0.075$ or 7.5%.

28. If \$1000 is invested at an interest rate $a\%$, then 2000 is invested at $\left(a + \frac{1}{2}\right)\%$, so, remembering that a is expressed as a percentage, the total interest is $I = 1000 \cdot \dfrac{a}{100} \cdot 1 + 2000 \cdot \dfrac{a + \frac{1}{2}}{100} \cdot 1 = 10a + 20a + 10 = 30a + 10$. Since the total interest is \$190, we have $190 = 30a + 10 \Leftrightarrow 180 = 30a \Leftrightarrow a = 6$. Thus, the \$1000 is invested at 6% interest.

29. Let x be her monthly salary. Since her annual salary $= 12 \times$ (monthly salary) $+$ (Christmas bonus) we have $97{,}300 = 12x + 8{,}500 \Leftrightarrow 88{,}800 = 12x \Leftrightarrow x \approx 7{,}400$. Her monthly salary is \$7,400.

30. Let s be the husband's annual salary. Then her annual salary is $1.15s$. Since husband's annual salary+wife's annual salary = total annual income, we have $s + 1.15s = 69{,}875 \Leftrightarrow 2.15s = 69{,}875 \Leftrightarrow s = 32{,}500$. Thus the husband's annual salary is \$32,500.

31. Let x be the overtime hours Helen works. Since gross pay = regular salary + overtime pay, we obtain the equation $352.50 = 7.50 \times 35 + 7.50 \times 1.5 \times x \Leftrightarrow 352.50 = 262.50 + 11.25x \Leftrightarrow 90 = 11.25x \Leftrightarrow x = \dfrac{90}{11.25} = 8$. Thus Helen worked 8 hours of overtime.

32. Let x be the hours the assistant worked. Then $2x$ is the hours the plumber worked. Since the labor charge is equal to the plumber's labor plus the assistant's labor, we have $4025 = 45\,(2x) + 25x \Leftrightarrow 4025 = 90x + 25x \Leftrightarrow 4025 = 115x \Leftrightarrow x = \frac{4025}{115} = 35$. Thus the assistant works for 35 hours, and the plumber works for $2 \times 35 = 70$ hours.

33. All ages are in terms of the daughter's age 7 years ago. Let y be age of the daughter 7 years ago. Then $11y$ is the age of the movie star 7 years ago. Today, the daughter is $y + 7$, and the movie star is $11y + 7$. But the movie star is also 4 times his daughter's age today. So $4(y + 7) = 11y + 7 \Leftrightarrow 4y + 28 = 11y + 7 \Leftrightarrow 21 = 7y \Leftrightarrow y = 3$. Thus the movie star's age today is $11(3) + 7 = 40$ years.

34. Let h be number of home runs Babe Ruth hit. Then $h + 41$ is the number of home runs that Hank Aaron hit. So $1469 = h + h + 41 \Leftrightarrow 1428 = 2h \Leftrightarrow h = 714$. Thus Babe Ruth hit 714 home runs.

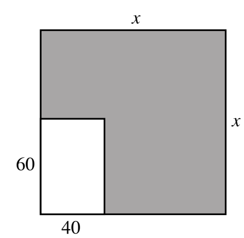
35. Let p be the number of pennies. Then p is the number of nickels and p is the number of dimes. So the value of the coins in the purse is the value of the pennies plus the value of the nickels plus the value of the dimes. Thus $1.44 = 0.01p + 0.05p + 0.10p \Leftrightarrow 1.44 = 0.16p \Leftrightarrow p = \frac{1.44}{0.16} = 9$. So the purse contains 9 pennies, 9 nickels, and 9 dimes.

36. Let q be the number of quarters. Then $2q$ is the number of dimes, and $2q + 5$ is the number of nickels. Thus $3.00 =$ value of the nickels$+$ value of the dimes$+$ value of the quarters. So
$3.00 = 0.05(2q + 5) + 0.10(2q) + 0.25q \Leftrightarrow 3.00 = 0.10q + 0.25 + 0.20q + 0.25q \Leftrightarrow 2.75 = 0.55q \Leftrightarrow q = \frac{2.75}{0.55} = 5$.
Thus Mary has 5 quarters, $2(5) = 10$ dimes, and $2(5) + 5 = 15$ nickels.

37. Let l be the length of the garden. Since area $=$ width $\cdot$ length, we obtain the equation $1125 = 25l \Leftrightarrow l = \frac{1125}{25} = 45$ ft. So the garden is 45 feet long.

38. Let w be the width of the pasture. Then the length of the pasture is $2w$. Since area$=$ length$\times$ width we have $115{,}200 = w(2w) = 2w^2 \Leftrightarrow w^2 = 57{,}600 \Rightarrow w = \pm 240$. Thus the width of the pasture is 240 feet.

39. Let x be the length of a side of the square plot. As shown in the figure, area of the plot $=$ area of the building $+$ area of the parking lot. Thus, $x^2 = 60(40) + 12{,}000 = 2{,}400 + 12{,}000 = 14{,}400 \Rightarrow x = \pm 120$. So the plot of land measures 120 feet by 120 feet.



40. Let w be the width of the building lot. Then the length of the building lot is $5w$. Since a half-acre is $\frac{1}{2} \cdot 43{,}560 = 21{,}780$ and area is length times width, we have $21{,}780 = w(5w) = 5w^2 \Leftrightarrow w^2 = 4{,}356 \Rightarrow w = \pm 66$. Thus the width of the building lot is 66 feet and the length of the building lot is $5(66) = 330$ feet.

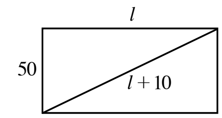
41. Let w be the width of the garden in feet. Then the length is $w + 10$. Thus $875 = w(w + 10) \Leftrightarrow w^2 + 10w - 875 = 0 \Leftrightarrow (w + 35)(w - 25) = 0$. So $w + 35 = 0$ in which case $w = -35$, which is not possible, or $w - 25 = 0$ and so $w = 25$. Thus the width is 25 feet and the length is 35 feet.

42. Let w be the width of the bedroom. Then its length is $w + 7$. Since area is length times width, we have $228 = (w + 7)w = w^2 + 7w \Leftrightarrow w^2 + 7w - 228 = 0 \Leftrightarrow (w + 19)(w - 12) = 0 \Leftrightarrow w + 19 = 0$ or $w - 12 = 0$. Thus $w = -19$ or $w = 12$. Since the width must be positive, the width is 12 feet.

43. Let w be the width of the garden in feet. We use the perimeter to express the length l of the garden in terms of width. Since the perimeter is twice the width plus twice the length, we have $200 = 2w + 2l \Leftrightarrow 2l = 200 - 2w \Leftrightarrow l = 100 - w$. Using the formula for area, we have $2400 = w(100 - w) = 100w - w^2 \Leftrightarrow w^2 - 100w + 2400 = 0 \Leftrightarrow (w - 40)(w - 60) = 0$. So $w - 40 = 0 \Leftrightarrow w = 40$, or $w - 60 = 0 \Leftrightarrow w = 60$. If $w = 40$, then $l = 100 - 40 = 60$. And if $w = 60$, then $l = 100 - 60 = 40$. So the length is 60 feet and the width is 40 feet.

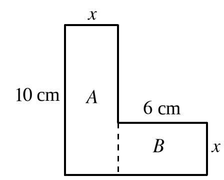
44. Let w be the width of the lot in feet. Then the length is $w + 6$. Using the Pythagorean Theorem, we have $w^2 + (w + 6)^2 = (174)^2 \Leftrightarrow w^2 + w^2 + 12w + 36 = 30{,}276 \Leftrightarrow 2w^2 + 12w - 30240 = 0 \Leftrightarrow w^2 + 6w - 15120 = 0 \Leftrightarrow (w + 126)(w - 120) = 0$. So either $w + 126 = 0$ in which case $w = -126$, which is not possible, or $w - 120 = 0$ in which case $w = 120$. Thus the width is 120 feet and the length is 126 feet.

45. Let l be the length of the lot in feet. Then the length of the diagonal is $l + 10$. We apply the Pythagorean Theorem with the hypotenuse as the diagonal. So $l^2 + 50^2 = (l + 10)^2 \Leftrightarrow l^2 + 2500 = l^2 + 20l + 100 \Leftrightarrow 20l = 2400 \Leftrightarrow l = 120$. Thus the length of the lot is 120 feet.

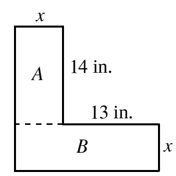


46. Let r be the radius of the running track. The running track consists of two semicircles and two straight sections 110 yards long, so we get the equation $2\pi r + 220 = 440 \Leftrightarrow 2\pi r = 220 \Leftrightarrow r = \frac{110}{\pi} = 35.03$. Thus the radius of the semicircle is about 35 yards.

47. **(a)** First we write a formula for the area of the figure in terms of x. Region A has dimensions 10 cm and x cm and region B has dimensions 6 cm and x cm. So the shaded region has area $(10 \cdot x) + (6 \cdot x) = 16x$ cm^2. We are given that this is equal to 144 cm^2, so $144 = 16x \Leftrightarrow x = \frac{144}{16} = 9$ cm.



(b) First we write a formula for the area of the figure in terms of x. Region A has dimensions 14 in. and x in. and region B has dimensions $(13 + x)$ in. and x in. So the area of the figure is $(14 \cdot x) + [(13 + x)\, x] = 14x + 13x + x^2 = x^2 + 27x$. We are given that this is equal to 160 in^2, so $160 = x^2 + 27x \Leftrightarrow x^2 + 27x - 160 = 0$ $\Leftrightarrow (x + 32)(x - 5) \Leftrightarrow x = -32$ or $x = 5$. x must be positive, so $x = 5$ in.



48. **(a)** The shaded area is the sum of the area of a square and the area of a triangle. So $A = y^2 + \frac{1}{2}(y)(y) = \frac{3}{2}y^2$. We are given that the area is 120 in^2, so $120 = \frac{3}{2}y^2 \Leftrightarrow y^2 = 80 \Leftrightarrow y = \pm\sqrt{80} = \pm 4\sqrt{5}$. y is positive, so $y = 4\sqrt{5} \approx 8.94$ in.

(b) The shaded area is the sum of the area of a rectangle and the area of a triangle. So $A = y(1) + \frac{1}{2}(y)(y) = \frac{1}{2}y^2 + y$. We are given that the area is 1200 cm^2, so $1200 = \frac{1}{2}y^2 + y \Leftrightarrow y^2 + 2y - 2400 = 0 \Leftrightarrow (y + 50)(y - 48) = 0$. y is positive, so $y = 48$ cm.

49. Let x be the width of the strip. Then the length of the mat is $20 + 2x$, and the width of the mat is $15 + 2x$. Now the perimeter is twice the length plus twice the width, so $102 = 2(20 + 2x) + 2(15 + 2x) \Leftrightarrow 102 = 40 + 4x + 30 + 4x \Leftrightarrow 102 = 70 + 8x \Leftrightarrow 32 = 8x \Leftrightarrow x = 4$. Thus the strip of mat is 4 inches wide.

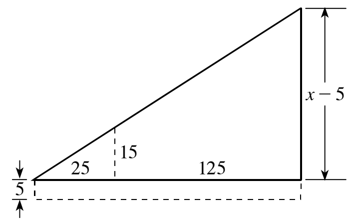
50. Let x be the width of the strip. Then the width of the poster is $100 + 2x$ and its length is $140 + 2x$. The perimeter of the printed area is $2(100) + 2(140) = 480$, and the perimeter of the poster is $2(100 + 2x) + 2(140 + 2x)$. Now we use the fact that the perimeter of the poster is $1\frac{1}{2}$ times the perimeter of the printed area: $2(100 + 2x) + 2(140 + 2x) = \frac{3}{2} \cdot 480 \Leftrightarrow$ $480 + 8x = 720 \Leftrightarrow 8x = 240 \Leftrightarrow x = 30$. The blank strip is thus 30 cm wide.

51. Let h be the height the ladder reaches (in feet). Using the Pythagorean Theorem we have $\left(7\frac{1}{2}\right)^2 + h^2 = \left(19\frac{1}{2}\right)^2 \Leftrightarrow$ $\left(\frac{15}{2}\right)^2 + h^2 = \left(\frac{39}{4}\right)^2 \Leftrightarrow h^2 = \left(\frac{39}{4}\right)^2 - \left(\frac{15}{2}\right)^2 = \frac{1521}{4} - \frac{225}{4} = \frac{1296}{4} = 324$. So $h = \sqrt{324} = 18$.

52. Let h be the height of the flagpole, in feet. Then the length of each guy wire is $h + 5$. Since the distance between the points where the wires are fixed to the ground is equal to one guy wire, the triangle is equilateral, and the flagpole is the perpendicular bisector of the base. Thus from the Pythagorean Theorem, we get $\left[\frac{1}{2}(h + 5)\right]^2 + h^2 = (h + 5)^2 \Leftrightarrow h^2 + 10h + 25 + 4h^2 = 4h^2 + 40h + 100 \Leftrightarrow h^2 - 30h - 75 = 0 \Rightarrow$ $h = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(1)(-75)}}{2(1)} = \frac{30 \pm \sqrt{900 + 300}}{2} = \frac{30 \pm \sqrt{1200}}{2} = \frac{30 \pm 20\sqrt{3}}{2}$. Since $h = \frac{30 - 20\sqrt{3}}{2} < 0$, we reject it. Thus the height is $h = \frac{30 + 20\sqrt{3}}{2} = 15 + 10\sqrt{3} \approx 32.32$ ft ≈ 32 ft 4 in.

53. Let x be the length of the man's shadow, in meters. Using similar triangles, $\dfrac{10+x}{6}=\dfrac{x}{2} \Leftrightarrow 20+2x=6x \Leftrightarrow 4x=20 \Leftrightarrow x=5$. Thus the man's shadow is 5 meters long.

54. Let x be the height of the tall tree. Here we use the property that corresponding sides in similar triangles are proportional. The base of the similar triangles starts at eye level of the woodcutter, 5 feet. Thus we obtain the proportion $\dfrac{x-5}{15}=\dfrac{150}{25} \Leftrightarrow 25(x-5)=15(150) \Leftrightarrow 25x-125=2250 \Leftrightarrow 25x=2375 \Leftrightarrow x=95$. Thus the tree is 95 feet tall.



55. Let x be the amount (in mL) of 60% acid solution to be used. Then $300-x$ mL of 30% solution would have to be used to yield a total of 300 mL of solution.

	60% acid	30% acid	Mixture
mL	x	$300-x$	300
Rate (% acid)	0.60	0.30	0.50
Value	$0.60x$	$0.30(300-x)$	$0.50(300)$

Thus the total amount of pure acid used is $0.60x+0.30(300-x)=0.50(300) \Leftrightarrow 0.3x+90=150 \Leftrightarrow x=\dfrac{60}{0.3}=200$. So 200 mL of 60% acid solution must be mixed with 100 mL of 30% solution to get 300 mL of 50% acid solution.

56. The amount of pure acid in the original solution is $300(50\%)=150$. Let x be the number of mL of pure acid added. Then the final volume of solution is $300+x$. Because its concentration is to be 60%, we must have $\dfrac{150+x}{300+x}=60\%=0.6 \Leftrightarrow 150+x=0.6(300+x) \Leftrightarrow 150+x=180+0.6x \Leftrightarrow 0.4x=30 \Leftrightarrow x=\dfrac{30}{0.4}=75$. Thus, 75 mL of pure acid must be added.

57. Let x be the number of grams of silver added. The weight of the rings is $5\times 18\text{ g}=90\text{ g}$.

	5 rings	Pure silver	Mixture
Grams	90	x	$90+x$
Rate (% gold)	0.90	0	0.75
Value	$0.90(90)$	$0x$	$0.75(90+x)$

So $0.90(90)+0x=0.75(90+x) \Leftrightarrow 81=67.5+0.75x \Leftrightarrow 0.75x=13.5 \Leftrightarrow x=\frac{13.5}{0.75}=18$. Thus 18 grams of silver must be added to get the required mixture.

58. Let x be the number of liters of water to be boiled off. The result will contain $6-x$ liters.

	Original	Water	Final
Liters	6	$-x$	$6-x$
Concentration	120	0	200
Amount	$120(6)$	0	$200(6-x)$

So $120(6)+0=200(6-x) \Leftrightarrow 720=1200-200x \Leftrightarrow 200x=480 \Leftrightarrow x=2.4$. Thus 2.4 liters need to be boiled off.

59. Let x be the number of liters of coolant removed and replaced by water.

	60% antifreeze	60% antifreeze (removed)	Water	Mixture
Liters	3.6	x	x	3.6
Rate (% antifreeze)	0.60	0.60	0	0.50
Value	0.60 (3.6)	$-0.60x$	$0x$	0.50 (3.6)

So $0.60\,(3.6) - 0.60x + 0x = 0.50\,(3.6) \Leftrightarrow 2.16 - 0.6x = 1.8 \Leftrightarrow -0.6x = -0.36 \Leftrightarrow x = \dfrac{-0.36}{-0.6} = 0.6$. Thus 0.6 liters must be removed and replaced by water.

60. Let x be the number of gallons of 2% bleach removed from the tank. This is also the number of gallons of pure bleach added to make the 5% mixture.

	Original 2%	Pure bleach	5% mixture
Gallons	$100 - x$	x	100
Concentration	0.02	1	0.05
Bleach	$0.02\,(100 - x)$	$1x$	$0.05 \cdot 100$

So $0.02\,(100 - x) + x = 0.05 \cdot 100 \Leftrightarrow 2 - 0.02x + x = 5 \Leftrightarrow 0.98x = 3 \Leftrightarrow x = 3.06$. Thus 3.06 gallons need to removed and replaced with pure bleach.

61. Let c be the concentration of fruit juice in the cheaper brand. The new mixture that Jill makes will consist of 650 mL of the original fruit punch and 100 mL of the cheaper fruit punch.

	Original Fruit Punch	Cheaper Fruit Punch	Mixture
mL	650	100	750
Concentration	0.50	c	0.48
Juice	$0.50 \cdot 650$	$100c$	$0.48 \cdot 750$

So $0.50 \cdot 650 + 100c = 0.48 \cdot 750 \Leftrightarrow 325 + 100c = 360 \Leftrightarrow 100c = 35 \Leftrightarrow c = 0.35$. Thus the cheaper brand is only 35% fruit juice.

62. Let x be the number of ounces of \$3.00/oz tea Then $80 - x$ is the number of ounces of \$2.75/oz tea.

	\$3.00 tea	\$2.75 tea	Mixture
Ounces	x	$80 - x$	80
Rate (cost per ounce)	3.00	2.75	2.90
Value	$3.00x$	$2.75\,(80 - x)$	2.90 (80)

So $3.00x + 2.75\,(80 - x) = 2.90\,(80) \Leftrightarrow 3.00x + 220 - 2.75x = 232 \Leftrightarrow 0.25x = 12 \Leftrightarrow x = 48$. The mixture uses 48 ounces of \$3.00/oz tea and $80 - 48 = 32$ ounces of \$2.75/oz tea.

63. Let t be the time in minutes it would take Candy and Tim if they work together. Candy delivers the papers at a rate of $\frac{1}{70}$ of the job per minute, while Tim delivers the paper at a rate of $\frac{1}{80}$ of the job per minute. The sum of the fractions of the job that each can do individually in one minute equals the fraction of the job they can do working together. So we have $\dfrac{1}{t} = \frac{1}{70} + \frac{1}{80} \Leftrightarrow 560 = 8t + 7t \Leftrightarrow 560 = 15t \Leftrightarrow t = 37\frac{1}{3}$ minutes. Since $\frac{1}{3}$ of a minute is 20 seconds, it would take them 37 minutes 20 seconds if they worked together.

64. Let t be the time, in minutes, it takes Hilda to mow the lawn. Since Hilda is twice as fast as Stan, it takes Stan $2t$ minutes to mow the lawn by himself. Thus $40 \cdot \dfrac{1}{t} + 40 \cdot \dfrac{1}{2t} = 1 \Leftrightarrow 40 + 20 = t \Leftrightarrow t = 60$. So it would take Stan $2\,(60) = 120$ minutes to mow the lawn.

65. Let t be the time, in hours, it takes Karen to paint a house alone. Then working together, Karen and Betty can paint a house in $\frac{2}{3}t$ hours. The sum of their individual rates equals their rate working together, so $\dfrac{1}{t}+\frac{1}{6}=\dfrac{1}{\frac{2}{3}t} \Leftrightarrow \dfrac{1}{t}+\frac{1}{6}=\dfrac{3}{2t} \Leftrightarrow$ $6+t=9 \Leftrightarrow t=3$. Thus it would take Karen 3 hours to paint a house alone.

66. Let h be the time, in hours, to fill the swimming pool using Jim's hose alone. Since Bob's hose takes 20% less time, it uses only 80% of the time, or $0.8h$. Thus $18\cdot\dfrac{1}{h}+18\cdot\dfrac{1}{0.8h}=1 \Leftrightarrow 18\cdot 0.8+18=0.8h \Leftrightarrow 14.4+18=0.8h \Leftrightarrow 32.4=0.8h$ $\Leftrightarrow h=40.5$. Jim's hose takes 40.5 hours, and Bob's hose takes 32.4 hours to fill the pool alone.

67. Let t be the time, in hours it takes Irene to wash all the windows. Then it takes Henry $t+\frac{3}{2}$ hours to wash all the windows, and the sum of the fraction of the job per hour they can do individually equals the fraction of the job they can do together. Since 1 hour 48 minutes $=1+\frac{48}{60}=1+\frac{4}{5}=\frac{9}{5}$, we have $\dfrac{1}{t}+\dfrac{1}{t+\frac{3}{2}.}=\dfrac{1}{\frac{9}{5}} \Leftrightarrow$ $\dfrac{1}{t}+\dfrac{2}{2t+3}=\frac{5}{9} \Rightarrow 9(2t+3)+2(9t)=5t(2t+3) \Leftrightarrow 18t+27+18t=10t^2+15t \Leftrightarrow 10t^2-21t-27=0$ $\Leftrightarrow t=\dfrac{-(-21)\pm\sqrt{(-21)^2-4(10)(-27)}}{2(10)}=\dfrac{21\pm\sqrt{441+1080}}{20}=\dfrac{21\pm 39}{20}$. So $t=\dfrac{21-39}{20}=-\dfrac{9}{10}$ or $t=\dfrac{21+39}{20}=3$. Since $t<0$ is impossible, all the windows are washed by Irene alone in 3 hours and by Henry alone in $3+\frac{3}{2}=4\frac{1}{2}$ hours.

68. Let t be the time, in hours, it takes Kay to deliver all the flyers alone. Then it takes Lynn $t+1$ hours to deliver all the flyers alone, and it takes the group $0.4t$ hours to do it together. Thus $\frac{1}{4}+\dfrac{1}{t}+\dfrac{1}{t+1}=\dfrac{1}{0.4t} \Leftrightarrow \frac{1}{4}(0.4t)+\dfrac{1}{t}(0.4t)+\dfrac{1}{t+1}(0.4t)=1$ $\Leftrightarrow t+4+\dfrac{4t}{t+1}=10 \Leftrightarrow t(t+1)+4(t+1)+4t=10(t+1) \Leftrightarrow t^2+t+4t+4+4t=10t+10 \Leftrightarrow t^2-t-6=0 \Leftrightarrow$ $(t-3)(t+2)=0$. So $t=3$ or $t=-2$. Since $t=-2$ is impossible, it takes Kay 3 hours to deliver all the flyers alone.

69. Let t be the time in hours that Wendy spent on the train. Then $\frac{11}{2}-t$ is the time in hours that Wendy spent on the bus. We construct a table:

	Rate	Time	Distance
By train	40	t	$40t$
By bus	60	$\frac{11}{2}-t$	$60\left(\frac{11}{2}-t\right)$

The total distance traveled is the sum of the distances traveled by bus and by train, so $300=40t+60\left(\frac{11}{2}-t\right) \Leftrightarrow$ $300=40t+330-60t \Leftrightarrow -30=-20t \Leftrightarrow t=\frac{-30}{-20}=1.5$ hours. So the time spent on the train is $5.5-1.5=4$ hours.

70. Let r be the speed of the slower cyclist, in mi/h. Then the speed of the faster cyclist is $2r$.

	Rate	Time	Distance
Slower cyclist	r	2	$2r$
Faster cyclist	$2r$	2	$4r$

When they meet, they will have traveled a total of 90 miles, so $2r+4r=90 \Leftrightarrow 6r=90 \Leftrightarrow r=15$. The speed of the slower cyclist is 15 mi/h, while the speed of the faster cyclist is $2(15)=30$ mi/h.

71. Let r be the speed of the plane from Montreal to Los Angeles. Then $r + 0.20r = 1.20r$ is the speed of the plane from Los Angeles to Montreal.

	Rate	Time	Distance
Montreal to L.A.	r	$\dfrac{2500}{r}$	2500
L.A. to Montreal	$1.2r$	$\dfrac{2500}{1.2r}$	2500

The total time is the sum of the times each way, so $9\frac{1}{6} = \dfrac{2500}{r} + \dfrac{2500}{1.2r} \Leftrightarrow \dfrac{55}{6} = \dfrac{2500}{r} + \dfrac{2500}{1.2r} \Leftrightarrow$ $55 \cdot 1.2r = 2500 \cdot 6 \cdot 1.2 + 2500 \cdot 6 \Leftrightarrow 66r = 18{,}000 + 15{,}000 \Leftrightarrow 66r = 33{,}000 \Leftrightarrow r = \frac{33{,}000}{66} = 500$. Thus the plane flew at a speed of 500 mi/h on the trip from Montreal to Los Angeles.

72. Let x be the speed of the car in mi/h. Since a mile contains 5280 ft and an hour contains 3600 s, 1 mi/h $= \frac{5280\text{ ft}}{3600\text{ s}} = \frac{22}{15}$ ft/s. The truck is traveling at $50 \cdot \frac{22}{15} = \frac{220}{3}$ ft/s. So in 6 seconds, the truck travels $6 \cdot \frac{220}{3} = 440$ feet. Thus the back end of the car must travel the length of the car, the length of the truck, and the 440 feet in 6 seconds, so its speed must be $\frac{14+30+440}{6} = \frac{242}{3}$ ft/s. Converting to mi/h, we have that the speed of the car is $\frac{242}{3} \cdot \frac{15}{22} = 55$ mi/h.

73. Let x be the rate, in mi/h, at which the salesman drove between Ajax and Barrington.

Cities	Distance	Rate	Time
Ajax $\to$ Barrington	120	x	$\dfrac{120}{x}$
Barrington $\to$ Collins	150	$x + 10$	$\dfrac{150}{x+10}$

We have used the equation time $= \dfrac{\text{distance}}{\text{rate}}$ to fill in the "Time" column of the table. Since the second part of the trip took 6 minutes (or $\frac{1}{10}$ hour) more than the first, we can use the time column to get the equation $\dfrac{120}{x} + \dfrac{1}{10} = \dfrac{150}{x+10} \Rightarrow$ $120\,(10)\,(x+10) + x\,(x+10) = 150\,(10x) \Leftrightarrow 1200x + 12{,}000 + x^2 + 10x = 1500x \Leftrightarrow x^2 - 290x + 12{,}000 = 0 \Leftrightarrow$ $x = \frac{-(-290)\pm\sqrt{(-290)^2-4(1)(12{,}000)}}{2} = \frac{290\pm\sqrt{84{,}100-48{,}000}}{2} = \frac{290\pm\sqrt{36{,}100}}{2} = \frac{290\pm190}{2} = 145 \pm 95$. Hence, the salesman drove either 50 mi/h or 240 mi/h between Ajax and Barrington. (The first choice seems more likely!)

74. Let x be the rate, in mi/h, at which Kiran drove from Tortula to Cactus.

Cities	Distance	Rate	Time
Tortula $\to$ Cactus	250	x	$\dfrac{250}{x}$
Cactus $\to$ Dry Junction	360	$x + 10$	$\dfrac{360}{x+10}$

We have used time $= \dfrac{\text{distance}}{\text{rate}}$ to fill in the time column of the table. We are given that the sum of the times is 11 hours. Thus we get the equation $\dfrac{250}{x} + \dfrac{360}{x+10} = 11 \Leftrightarrow 250\,(x+10) + 360x =$ $11x\,(x+10) \Leftrightarrow 250x + 2500 + 360x = 11x^2 + 110x \Leftrightarrow 11x^2 - 500x - 2500 = 0 \Rightarrow$ $x = \dfrac{-(-500) \pm \sqrt{(-500)^2 - 4\,(11)\,(-2500)}}{2\,(11)} = \dfrac{500 \pm \sqrt{250{,}000 + 110{,}000}}{22} = \dfrac{500 \pm \sqrt{360{,}000}}{22} = \dfrac{500 \pm 600}{22}$. Hence, Kiran drove either -4.54 mi/h (impossible) or 50 mi/h between Tortula and Cactus.

75. Let r be the rowing rate in km/h of the crew in still water. Then their rate upstream was $r - 3$ km/h, and their rate downstream was $r + 3$ km/h.

	Distance	Rate	Time
Upstream	6	$r - 3$	$\frac{6}{r-3}$
Downstream	6	$r + 3$	$\frac{6}{r+3}$

Since the time to row upstream plus the time to row downstream was 2 hours 40 minutes $= \frac{8}{3}$ hour, we get the equation $\frac{6}{r-3} + \frac{6}{r+3} = \frac{8}{3} \Leftrightarrow 6(3)(r+3) + 6(3)(r-3) = 8(r-3)(r+3) \Leftrightarrow 18r + 54 + 18r - 54 = 8r^2 - 72 \Leftrightarrow$ $0 = 8r^2 - 36r - 72 = 4\left(2r^2 - 9r - 18\right) = 4(2r+3)(r-6)$. Since $2r + 3 = 0 \Leftrightarrow r = -\frac{3}{2}$ is impossible, the solution is $r - 6 = 0 \Leftrightarrow r = 6$. So the rate of the rowing crew in still water is 6 km/h.

76. Let r be the speed of the southbound boat. Then $r + 3$ is the speed of the eastbound boat. In two hours the southbound boat has traveled $2r$ miles and the eastbound boat has traveled $2(r+3) = 2r + 6$ miles. Since they are traveling is directions with are 90° apart, we can use the Pythagorean Theorem to get $(2r)^2 + (2r+6)^2 = 30^2 \Leftrightarrow 4r^2 + 4r^2 + 24r + 36 = 900$ $\Leftrightarrow 8r^2 + 24r - 864 = 0 \Leftrightarrow 8\left(r^2 + 3r - 108\right) = 0 \Leftrightarrow 8(r+12)(r-9) = 0$. So $r = -12$ or $r = 9$. Since speed is positive, the speed of the southbound boat is 9 mi/h.

77. Let x be the distance from the fulcrum to where the mother sits. Then substituting the known values into the formula given, we have $100(8) = 125x \Leftrightarrow 800 = 125x \Leftrightarrow x = 6.4$. So the mother should sit 6.4 feet from the fulcrum.

78. Let w be the largest weight that can be hung. In this exercise, the edge of the building acts as the fulcrum, so the 240 lb man is sitting 25 feet from the fulcrum. Then substituting the known values into the formula given in Exercise 43, we have $240(25) = 5w \Leftrightarrow 6000 = 5w \Leftrightarrow w = 1200$. Therefore, 1200 pounds is the largest weight that can be hung.

79. We have that the volume is 180 ft^3, so $x(x-4)(x+9) = 180 \Leftrightarrow x^3 + 5x^2 - 36x = 180 \Leftrightarrow x^3 + 5x^2 - 36x - 180 = 0$ $\Leftrightarrow x^2(x+5) - 36(x+5) = 0 \Leftrightarrow (x+5)\left(x^2 - 36\right) = 0 \Leftrightarrow (x+5)(x+6)(x-6) = 0 \Rightarrow x = 6$ is the only positive solution. So the box is 2 feet by 6 feet by 15 feet.

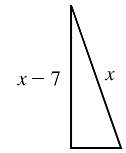
80. Let r be the radius of the larger sphere, in mm. Equating the volumes, we have $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(2^3 + 3^3 + 4^3\right) \Leftrightarrow$ $r^3 = 2^3 + 3^3 + 4^4 \Leftrightarrow r^3 = 99 \Leftrightarrow r = \sqrt[3]{99} \approx 4.63$. Therefore, the radius of the larger sphere is about 4.63 mm.

81. Let x be the length of one side of the cardboard, so we start with a piece of cardboard x by x. When 4 inches are removed from each side, the base of the box is $x - 8$ by $x - 8$. Since the volume is 100 in^3, we get $4(x-8)^2 = 100 \Leftrightarrow$ $x^2 - 16x + 64 = 25 \Leftrightarrow x^2 - 16x + 39 = 0 \Leftrightarrow (x-3)(x-13) = 0$.So $x = 3$ or $x = 13$. But $x = 3$ is not possible, since then the length of the base would be $3 - 8 = -5$, and all lengths must be positive. Thus $x = 13$, and the piece of cardboard is 13 inches by 13 inches.

82. Let r be the radius of the can. Now using the formula $V = \pi r^2 h$ with $V = 40\pi$ cm^3 and $h = 10$, we solve for r. Thus $40\pi = \pi r^2(10) \Leftrightarrow 4 = r^2 \Rightarrow r = \pm 2$. Since r represents radius, $r > 0$. Thus $r = 2$, and the diameter is 4 cm.

83. Let r be the radius of the tank, in feet. The volume of the spherical tank is $\frac{4}{3}\pi r^3$ and is also $750 \times 0.1337 = 100.275$. So $\frac{4}{3}\pi r^3 = 100.275 \Leftrightarrow r^3 = 23.938 \Leftrightarrow r = 2.88$ feet.

84. Let x be the length of the hypotenuse of the triangle, in feet. Then one of the other sides has length $x-7$ feet, and since the perimeter is 392 feet, the remaining side must have length $392-x-(x-7)=399-2x$. From the Pythagorean Theorem, we get $(x-7)^2+(399-2x)^2=x^2 \Leftrightarrow 4x^2-1610x+159250=0$. Using the Quadratic Formula, we get



$x=\frac{1610\pm\sqrt{1610^2-4(4)(159250)}}{2(4)}=\frac{1610\pm\sqrt{44100}}{8}=\frac{1610\pm210}{8}$, and so $x=227.5$ or $x=175$. But if $x=227.5$, then the side of length $x-7$ combined with the hypotenuse already exceeds the perimeter of 392 feet, and so we must have $x=175$. Thus the other sides have length $175-7=168$ and $399-2(175)=49$. The lot has sides of length 49 feet, 168 feet, and 175 feet.

85. Let x be the length, in miles, of the abandoned road to be used. Then the length of the abandoned road not used is $40-x$, and the length of the new road is $\sqrt{10^2+(40-x)^2}$ miles, by the Pythagorean Theorem. Since the cost of the road is cost per mile $\times$ number of miles, we have $100{,}000x+200{,}000\sqrt{x^2-80x+1700}=6{,}800{,}000$ $\Leftrightarrow 2\sqrt{x^2-80x+1700}=68-x$.Squaring both sides, we get $4x^2-320x+6800=4624-136x+x^2 \Leftrightarrow$ $3x^2-184x+2176=0 \Leftrightarrow x=\frac{184\pm\sqrt{33856-26112}}{6}=\frac{184\pm88}{6} \Leftrightarrow x=\frac{136}{3}$ or $x=16$. Since $45\frac{1}{3}$ is longer than the existing road, 16 miles of the abandoned road should be used. A completely new road would have length $\sqrt{10^2+40^2}$ (let $x=0$) and would cost $\sqrt{1700}\times200{,}000\approx8.3$ million dollars. So no, it would not be cheaper.

86. Let x be the distance, in feet, that he goes on the boardwalk before veering off onto the sand. The distance along the boardwalk from where he started to the point on the boardwalk closest to the umbrella is $\sqrt{750^2-210^2}=720$ ft. Thus the distance that he walks on the sand is $\sqrt{(720-x)^2+210^2}=\sqrt{518{,}400-1440x+x^2+44{,}100}=\sqrt{x^2-1440x+562{,}500}$.

	Distance	Rate	Time
Along boardwalk	x	4	$\frac{x}{4}$
Across sand	$\sqrt{x^2-1440x+562{,}500}$	2	$\frac{\sqrt{x^2-1440x+562{,}500}}{2}$

Since 4 minutes 45 seconds $=$ 285 seconds, we equate the time it takes to walk along the boardwalk and across the sand to the total time to get $285=\frac{x}{4}+\frac{\sqrt{x^2-1440x+562{,}500}}{2} \Leftrightarrow 1140-x=2\sqrt{x^2-1440x+562{,}500}$. Squaring both sides, we get $(1140-x)^2=4\left(x^2-1440x+562{,}500\right) \Leftrightarrow 1{,}299{,}600-2280x+x^2=4x^2-5760x+2{,}250{,}000$ $\Leftrightarrow 0=3x^2-3480x+950{,}400=3\left(x^2-1160x+316{,}800\right)=3(x-720)(x-440)$. So $x-720=0$ $\Leftrightarrow x=720$, and $x-440=0 \Leftrightarrow x=440$. Checking $x=720$, the distance across the sand is 210 feet. So $\frac{720}{4}+\frac{210}{2}=180+105=285$ seconds. Checking $x=440$, the distance across the sand is $\sqrt{(720-440)^2+210^2}=350$ feet. So $\frac{440}{4}+\frac{350}{2}=110+175=285$ seconds. Since both solutions are less than or equal to 720 feet, we have two solutions: he walks 440 feet down the boardwalk and then heads towards his umbrella, or he walks 720 feet down the boardwalk and then heads toward his umbrella.

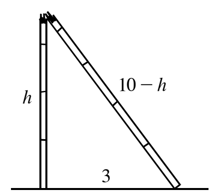
87. Let x be the height of the pile in feet. Then the diameter is $3x$ and the radius is $\frac{3}{2}x$ feet. Since the volume of the cone is 1000 ft^3, we have $\frac{\pi}{3}\left(\frac{3x}{2}\right)^2x=1000 \Leftrightarrow \frac{3\pi x^3}{4}=1000 \Leftrightarrow x^3=\frac{4000}{3\pi} \Leftrightarrow x=\sqrt[3]{\frac{4000}{3\pi}}\approx7.52$ feet.

88. Let h be the height of the screens in inches. The width of the smaller screen is $h+7$ inches, and the width of the bigger screen is $1.8h$ inches. The diagonal measure of the smaller screen is $\sqrt{h^2+(h+7)^2}$, and the diagonal measure of the larger screen is $\sqrt{h^2+(1.8h)^2}=\sqrt{4.24h^2}\approx 2.06h$. Thus $\sqrt{h^2+(h+7)^2}+3=2.06h \Leftrightarrow \sqrt{h^2+(h+7)^2}=2.06h-3$. Squaring both sides gives $h^2+h^2+14h+49=4.24h^2-12.36h+9 \Leftrightarrow 0=2.24h^2-26.36h-40$. Applying the Quadratic Formula, we obtain $h=\frac{26.36\pm\sqrt{(-26.36)^2-4(2.24)(-40)}}{2(2.24)}=\frac{26.36\pm\sqrt{1053.2496}}{4.48}\approx\dfrac{26.36\pm 32.45}{4.48}$. So $h\approx\dfrac{26.36\pm 32.45}{4.48}\approx 13.13$. Thus, the screens are approximately 13.1 inches high.

89. Let h be the height in feet of the structure. The structure is composed of a right cylinder with radius 10 and height $\frac{2}{3}h$ and a cone with base radius 10 and height $\frac{1}{3}h$. Using the formulas for the volume of a cylinder and that of a cone, we obtain the equation $1400\pi=\pi(10)^2\left(\frac{2}{3}h\right)+\frac{1}{3}\pi(10)^2\left(\frac{1}{3}h\right)\Leftrightarrow 1400\pi=\frac{200\pi}{3}h+\frac{100\pi}{9}h\Leftrightarrow 126=6h+h$ (multiply both sides by $\dfrac{9}{100\pi}$) $\Leftrightarrow 126=7h\Leftrightarrow h=18$. Thus the height of the structure is 18 feet.

90. Let y be the circumference of the circle, so $360-y$ is the perimeter of the square. Use the circumference to find the radius, r, in terms of y: $y=2\pi r\Rightarrow r=y/(2\pi)$. Thus the area of the circle is $\pi\left[y/(2\pi)\right]^2=y^2/(4\pi)$. Now if the perimeter of the square is $360-y$, the length of each side is $\frac{1}{4}(360-y)$, and the area of the square is $\left[\frac{1}{4}(360-y)\right]^2$. Setting these areas equal, we obtain $y^2/(4\pi)=\left[\frac{1}{4}(360-y)\right]^2\Leftrightarrow y/\left(2\sqrt{\pi}\right)=\frac{1}{4}(360-y)\Leftrightarrow 2y=360\sqrt{\pi}-\sqrt{\pi}y$ $\Leftrightarrow\left(2+\sqrt{\pi}\right)y=360\sqrt{\pi}$. Therefore, $y=360\sqrt{\pi}/\left(2+\sqrt{\pi}\right)\approx 169.1$. Thus one wire is 169.1 in. long and the other is 190.9 in. long.

91. Let h be the height of the break, in feet. Then the portion of the bamboo above the break is $10-h$. Applying the Pythagorean Theorem, we obtain $h^2+3^2=(10-h)^2\Leftrightarrow h^2+9=100-20h+h^2\Leftrightarrow -91=-20h\Leftrightarrow$ $h=\frac{91}{20}=4.55$. Thus the break is 4.55 ft above the ground.



92. Pythagoras was born about 569 BC in Samos, Ionia and died about 475 BC.
Euclid was born about 325 BC and died about 265 BC in Alexandria, Egypt.
Archimedes was born in 287 BC in Syracuse, Sicily and died in 212 BC in Syracuse, Sicily.

93. Answers will vary.

94. Let x equal the original length of the reed in cubits. Then $x-1$ is the piece that fits 60 times along the length of the field, that is, the length is $60(x-1)$. The width is $30x$. Then converting cubits to ninda, we have $375=60(x-1)\cdot 30x\cdot\frac{1}{12^2}=\frac{25}{2}x(x-1)\Leftrightarrow 30=x^2-x\Leftrightarrow x^2-x-30=0\Leftrightarrow(x-6)(x+5)=0$. So $x=6$ or $x=-5$. Since x must be positive, the original length of the reed is 6 cubits.

1.8 INEQUALITIES

1. **(a)** If $x<5$, then $x-3<5-3\Rightarrow x-3<2$.
(b) If $x\le 5$, then $3\cdot x\le 3\cdot 5\Rightarrow 3x\le 15$.
(c) If $x\ge 2$, then $-3\cdot x\le -3\cdot 2\Rightarrow -3x\le -6$.
(d) If $x<-2$, then $-x>2$.